The noise and its effects on detector signal processing

Pier Francesco Manfredi

pfmanfredi@lbl.gov

Fundamental considerations about noise

The noise in electron devices, especially those belonging to the first amplification circuit, where the signal has the smallest amplitude, if not accurately controlled, may seriously degrade the quality of the measurements done with radiation detectors.

It must be pointed out that in the technical literature the word **NOISE** is frequently, though improperly, used to describe different categories of phenomena, all contributing to affect the behavior of the circuits on small signals.

For instance, the word **NOISE** is frequently used to refer to the interference from sources external to the detector and its associated measuring system and this, more correctly, should be referred to as environment noise (pick-up noise, ground-loop noise and so on). It is possible to think, at least ideally, that this type of noise is removed by skillful grounding and shielding techniques.

Sometimes the word **NOISE** is also used to refer to the interference from sources internal to the detector signal processor, basically the large switching signals in its digital section. Still in an ideal way of thinking, it is possible to assume that this type of noise is cancelled by highly skilled layouting techniques.

Even if the previous precautions are fulfilled, some smearing is still observed on the signal arriving from the detector. Such a smearing is due to the *really fundamental noise*, which is something associated with the physical processes that take place in the electron devices.

To distinguish it from the two previous types of noise, this noise should be referred to as ***purely stochastic noise***, for it is of random nature and is related to spontaneous fluctuations of matter and electricity and cannot be eliminated by engineering techniques.

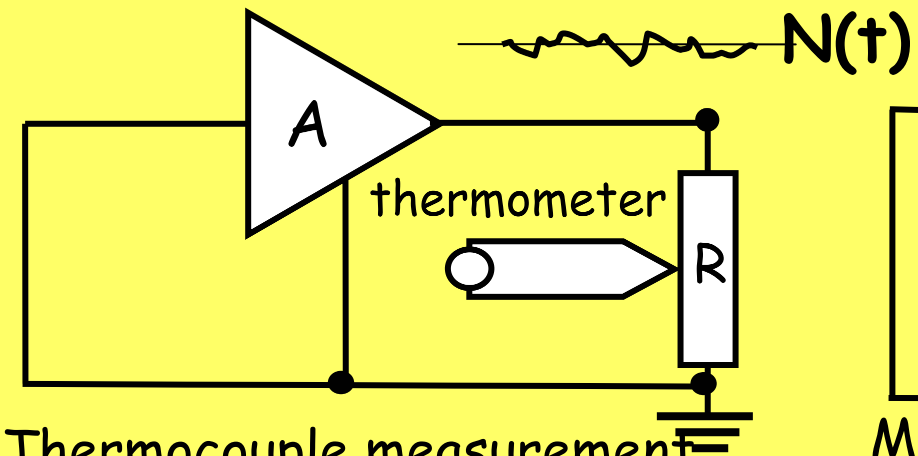
In the statistical language, the noise can be defined a continuous stochastic process with a continuous parameter, the time and represented as { N(t), t }. Owing to its nature of random process, the noise cannot be described by a deterministic time dependence.
In the usual applications it is sufficient to operate on the following quantities:

- The average $\langle N(t) \rangle$
- The root mean square value $\langle N(t)^2 \rangle$
- The variance $\sigma^2 = \langle N(t)^2 \rangle - \langle N(t) \rangle^2$

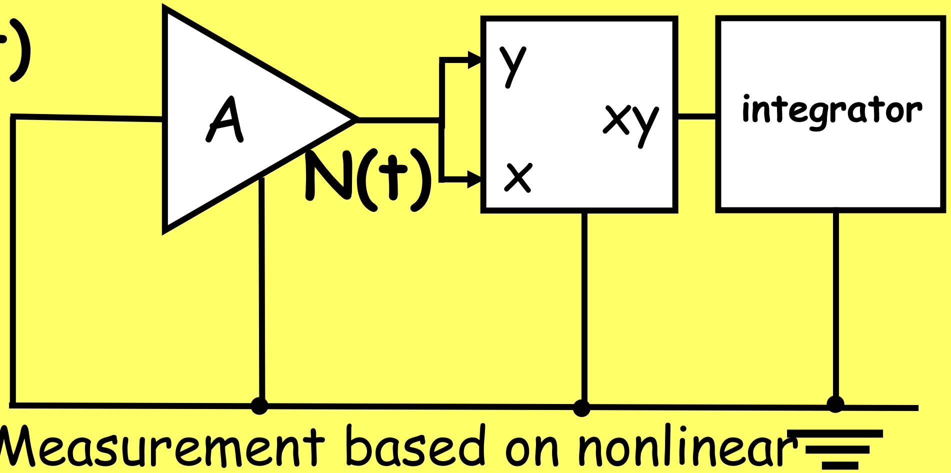
In most of applications the noise considered will have zero average, in which case the root mean square value of the process coincides with its variance. In the case of a noise of zero average and of stationary nature, that is, a noise whose statistical properties do not change in time, $\sigma^2 = \langle N(t)^2 \rangle$ is defined by the following limit (if it exists) on the truncated signal:

$$\sigma^2 = \langle N(t)^2 \rangle = \lim_{T \to \infty} (1/2T) \int_{-T}^{+T} N(t)^2 dt$$

Noise measurements

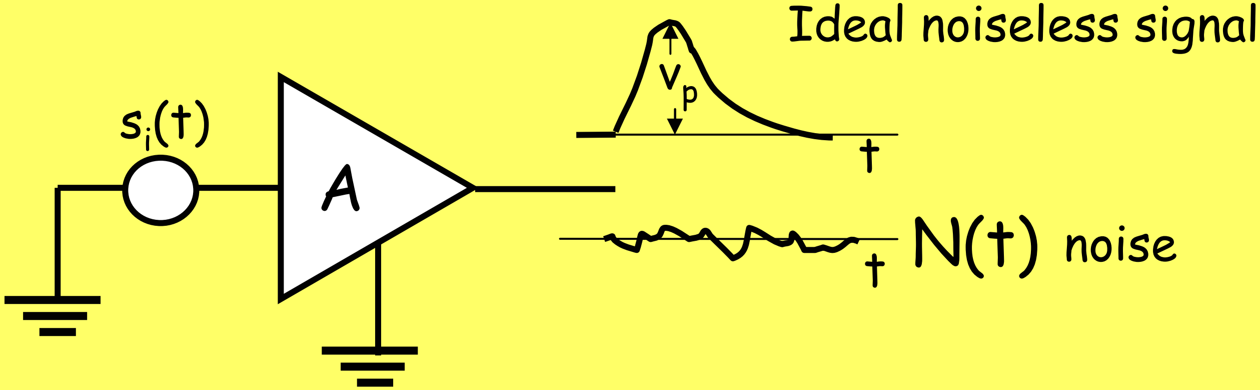


Thermocouple measurement



Measurement based on nonlinear analog processing

SIGNAL/NOISE RATIO



Suppose now that the signal $s_i(t)$, applied at the amplifier input appears at the output with a peak amplitude V_p. The signal/noise ratio in the measurement of the peak amplitude is defined as:

$\eta = V_p / \sigma$ and for a noise of zero average $\eta = V_p / \langle N(t)^2 \rangle^{1/2}$

Next step is the evaluation of the root mean square value of the noise at the output of a linear network. For this purpose it is necessary to discuss the noise representations. The noise will always be considered of stationary nature.

A fundamental concept for noise calculation is the spectral power density of the noise, defined as the function *S(f)* which, multiplied by the frequency interval df yields the elementary contribution brought about to the root mean square value $\langle N(t)^2 \rangle$ by the spectral components in the frequency interval f, f+df:

$$d\langle N^2\rangle/df = S(f)$$

In what follows the bilateral representation of the spectral power density will be adopted, with the frequency ranging in the interval

$$-\infty < f < +\infty$$

The physical dimensions of S(f) depend on the nature of the noise N. So, if N is

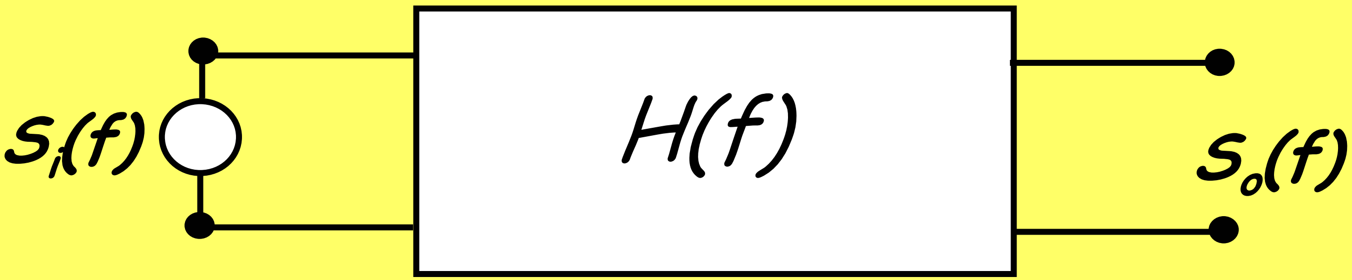
- a voltage, S(f) si expressed in V^2/Hz
- a current, S(f) is expressed in A^2/Hz

The plot of S(f) as a function of f is the noise spectrum. Its knowledge is essential in the characterization of the noise properties of electron devices. Instruments, called SPECTRUM ANALYZERS are used to determine the frequency dependence of the noise power density in electron devices under investigation.

If the spectral density S(f) is filtered in the finite frequency interval f_1 - f_2 the root mean square value contribution due to the frequencies in the transmitted band is calculated by integrating S(f) between f_1 and f_2 :

$$\langle N(t)^2 \rangle_{f1,f2} = \int_{f_1}^{f_2} S(f)\, df$$

The transformation of the spectral density $S_i(f)$ by a una linear network described by the transfer function in sinusoidal waves *H(f)* is given by:



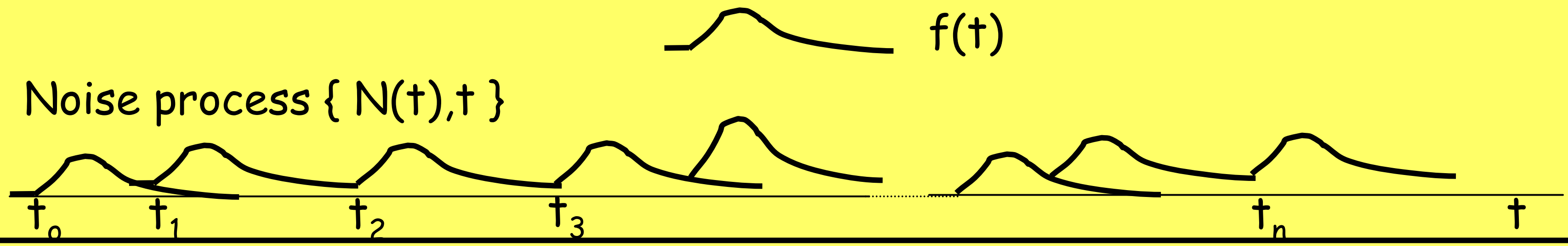
$$S_o(f) = S_i(f) \times |H(f)|^2$$

The root mean square noise at the output of the network is calculated as:

$$\langle N(t)^2 \rangle_o = \int_{-\infty}^{+\infty} S_o(f)\,df = \int_{-\infty}^{+\infty} S_i(f) \times |H(f)|^2 df$$

NOISE REPRESENTATION IN THE TIME DOMAIN: CAMPBELL's THEOREM ***(See appendix for more detail)***

A noise process can be interpreted in the time domain by virtue of the following Campbell's theorem. Consider a square-integrable signal f(t). Starting from f(t) a random process can be built-up as a sequence of identical signals f(t) that occur at random instants, distributed in time according to a poissonian law.



Campbell's theorem states that, calling λ the mean rate of arrival, the variance of the noise process is given by:

$$\sigma^2 = \langle N(t)^2\rangle - \langle N(t)\rangle^2 = \lambda \int_{-\infty}^{+\infty} f^2(t)\,dt = \lambda \int_{-\infty}^{+\infty} \left|\Im(f)\right|^2 df = \int_{-\infty}^{+\infty} S_N(f)\,df$$

where $S_N(f)$ is the spectral power density ot the N(t) process and $\Im(f)$ the Fourier transform di f(t). According to the last relationship:

$$S_N(f) = \lambda \left|\Im(f)\right|^2$$

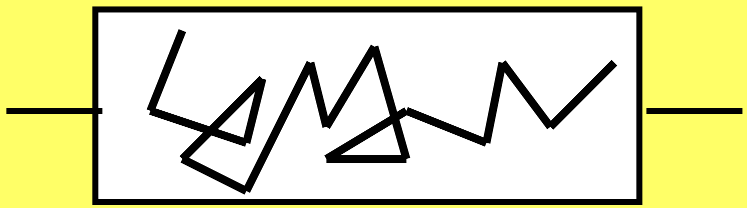
Noise sources in electron devices

This section is devoted to a basic discussion of the phenomena that are responsible for the presence of noise in the electron devices that are more commonly employed in the design of circuits associate with radiation detectors.
After describing the noise sources in passive devices, the attention will be focused on the noise processes that take place in the following active devices:

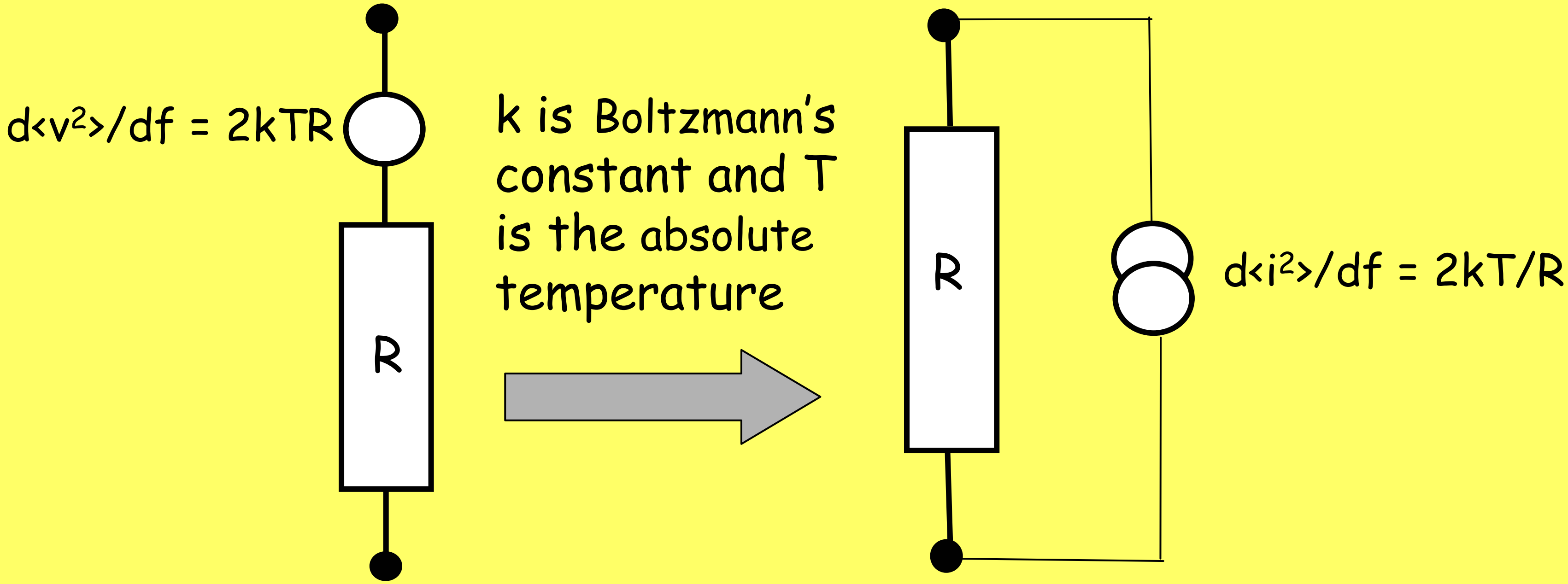
- Bipolar transistor (Si and Si-Ge BJT)
- MOSFETs (either enhancement or depletion-type devices)
- Silicon junction-field effect transistor (Si JFET)
- Ga As MESFET

Detectors featuring a high number of sensitive electrodes has led to the need of monolithic circuits based on some of the devices above.

Thermal noise. The first measurements on thermal noise, due to Johnson were done on metal resistors. The theoretical interpretation of thermal noise is due to Nyquist. The thermal noise in a metal resistor was explained as due to the thermal agitation of the electrons. The electrons describe erratic trajectories resulting from the combined effects of thermal agitation and scattering by the ionized atoms in the crystal lattice (see figure below).



Assuming a Maxwell-Boltzmann statistics for the electron gas in the metal, the thermal noise in a metal resistor can be represented by a noise voltage source with a spectral power density $d\langle v^2\rangle/df = 2kTR$ in series with the resistor or by a noise current source with a spectral power density $d\langle i^2\rangle/df = 2kT/R$ in parallel to the resistor. In the previous expressions k is Boltzmann's constant, $k=1.38\times 10^{-23}$ Joule/degree Kelvin and T is the absolute temperature. The thermal noise representations are shown in the next page.



- The thermal noise is related to the dissipative nature of the resistor
- Any resistor at thermal equilibrium, regardless of its nature, presents only thermal noise
- When current flows through a metallic resistor, it features only thermal noise and this is largely independent of the current value, while a nonmetallic resistor features in general additional (excess) noise

Thermal noise is related to the dissipative nature of the resistor.

Ideal capacitors and ideal inductors, that do not dissipate energy, but just store it and release it, do not feature thermal noise and they can be considered ***noiseless***.

Real capacitors and real inductors, however, are not noiseless and their noise is related to the losses, either of dielectric nature in a nonideal insulator in a capacitor and of magnetic nature in a nonideal material in the core of an inductor.

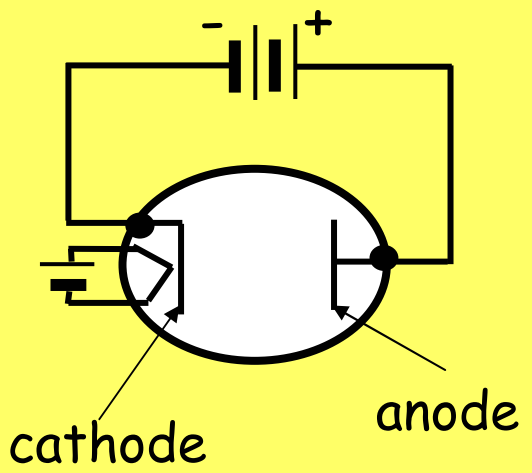
Moving to active devices, different mechanisms govern the noise in the high frequency region and in the low-frequency region. The noise in the **high frequency region**, which is of higher importance in detector applications can be interpreted on the basis of the following three models:

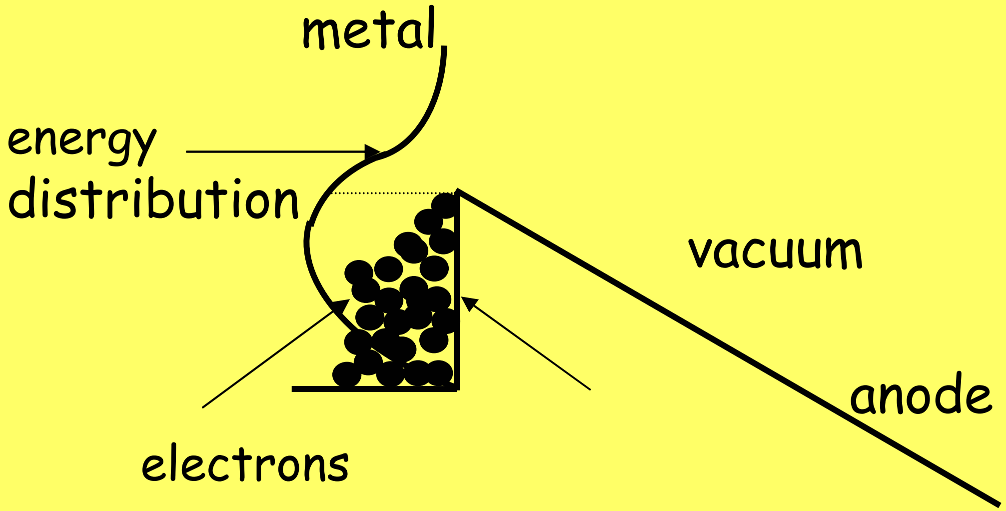
- thermal noise
- shot noise
- partition noise

Shot noise and partition noise are the basis for understanding the high frequency noise in the bipolar transistors.

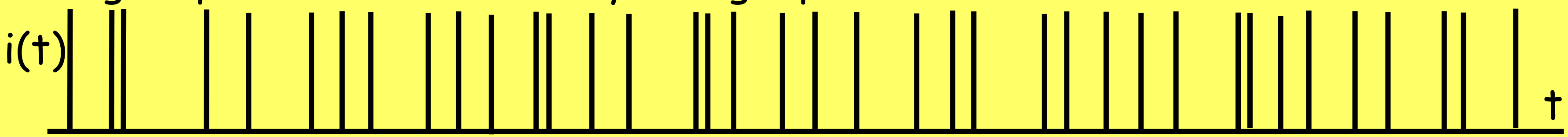
Thermal noise provides the interpretation of the high frequency noise in the channel of field-effect devices, JFETs, MOSFETs and MESFETs. It helps also understand the noise in the gate current of these devices at moderately high frequencies. **Shot noise** explains the gate current noise in Si JFET and GaAs MESFETs at low frequencies

Shot Noise - First analyzed in a vacuum diode operating in condition of saturation (absence of space charge in the proximity of the emitting cathode), its theory was formulated by Schottky.





The shot effect is related to the granular structure of electricity. In a microscopic observation the current in the diode would appear as consisting of elementary pulses randomly distributed in time, each inducing on the anode a charge equal to the elementary charge $q=1.6\times10^{-19}C$.

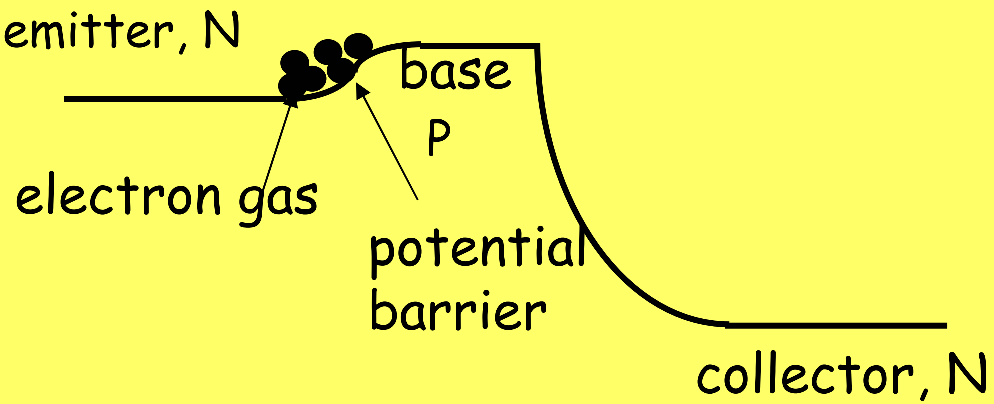


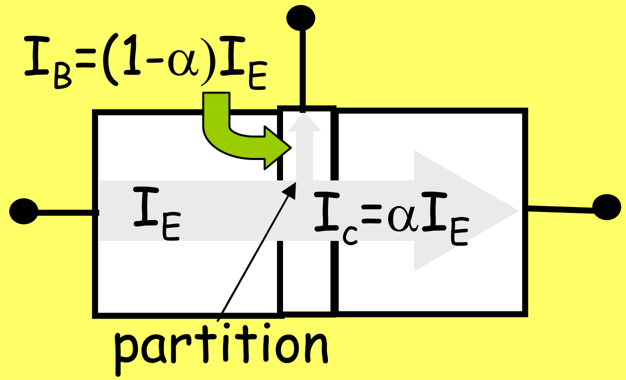
The current representation of the previous page is done under the hypothesis of a negligible transit time, because of which the individual induced-current contributions are assumed to be delta-impulses. As the process of electron emission by the **cathode** by thermoionic effect is of random nature, the number of electrons arriving at the anode in equal time intervals fluctuates from time interval to time interval and these fluctuations constitute the shot noise.
Shot noise belongs to the type of processes studied with Campbell's theorem. In this case the frequency λ is the standing current I in the diode divided by the elementary charge q: $\lambda = I/q$. The function f(t) is $q\delta$ and its Fourier transform is q. It is concluded that the spectral power density of shot noise is:

$$\lambda q^2 = (I/q)\, q^2 = qI$$

The noise is represented as a current source in parallel to the diode. The shot noise described by the spectral density above is named in the literature *full shot noise* to distinguish it from the so called *suppressed shot noise,* described by the spectral density $\langle di^2 \rangle/df = qF^2 I$, $F^2<1$ which would featured by a diode operating in conditions of space charge.

Noise in bipolar transistors





The gas of electrons in the highly doped emitter region is in conditions of thermal agitation. The potential barrier at the base-emitter junction actuates on the electrons of the emitter gas an energy selection. This situation is reminiscent of the process occurring in a saturated diode, which explains why the emitter current in a bipolar transistor exhibits full shot noise.

The emitter current splits between base and collector, the larger fraction α reaching the collector and the remainder $1-\alpha$ flowing to the base. In reality the current division is a random process and α can be assumed as the probability that an electron from the emitter reach the collector. Therefore partition noise occurs as the emitter current splits between base and collector. *(See appendix)*

The noise associated with the emitter current is therefore:

$$d\langle i_E^2\rangle/df = qI_E$$

A rigid partition of the emitter current between base (fraction 1- α) and collector (fraction α) would yield the following expressions for the spectral power densities of the noise associated with the base and collector currents:

$$d\langle i_B^2\rangle/df = qI_E\,(1-\alpha)^2 \qquad\qquad d\langle i_c2\rangle/df = qI_E\alpha^2$$

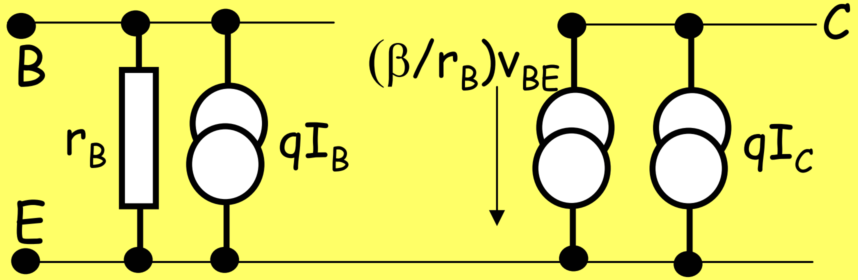
The partition noise results in the term $qI_E\,\alpha(1-\alpha)$ added to both spectral densities, that therefore become:

$$d\langle i_B^2\rangle/df = qI_E(1-\alpha)^2 + qI_E\alpha(1-\alpha) = qI_B$$

$$d\langle i_c^2\rangle/df = qI_E\,\alpha^2 + qI_E\alpha(1-\alpha) = qIc$$

The equivalent circuit of the bipolar transistor including the noise sources is given in the next page.

The equivalent circuit represented here is an oversimplified version of the hybrid π schematic to which the two noise sources of the previous discussion have been added.

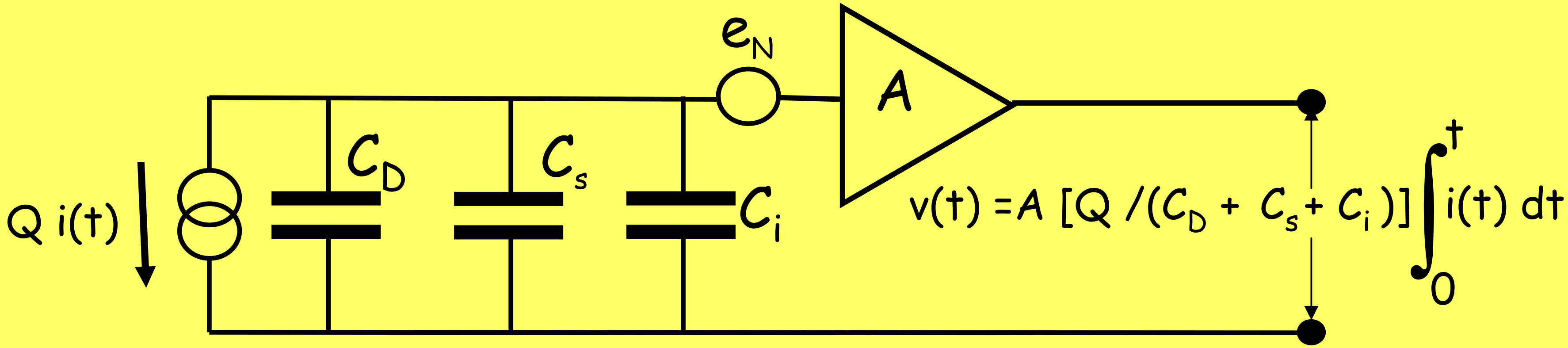


In this circuit the capital letters represent the standing components of the currents, while the lower case symbols refer to the signal components or to the incremental values.

In the equivalent circuit $r_B = kT/qI_B$ where k is Boltzmann's constant T is the absolute temperature q the electron charge and $\beta = \alpha/(1-\alpha)$.

The noise sources discussed so far are intimately related to the the physical processes that take place in the bipolar transistor. There is one more noise source which, though of extrinsic nature may represent an important limitation.

This approach would make sense only if the detector capacitance is linear, that is, voltage-independent and reliable in value. Even if these conditions are met, however, the voltage signal appearing across the detector capacitance would be too small to be processed by the circuits that follow. Amplification is therefore required, as shown in the figure below.

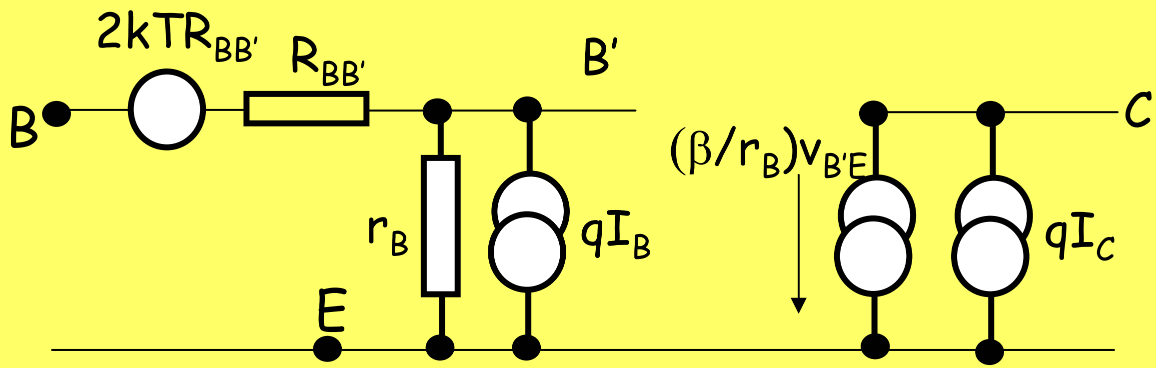


C_s represents the total stray capacitance which appears between the detector terminals

C_i is the input capacitance of the amplifier.

A is the gain of the amplifier, which is represented as a noiseless block. Its noisy nature is accounted for by the input-referred voltage noise source e_N

This is the thermal noise associated with the base spreading resistance of the bipolar transistor. The best discrete bipolar transistors intended for low-noise applications feature values of the base spreading resistance of a very few Ohm.



In such a case the effect of $R_{BB'}$ may be negligible so in the signal response calculations it is possible to replace R_{BB}' by a short circuit, while its noise contribution is still accounted for by the thermal noise series voltage source.

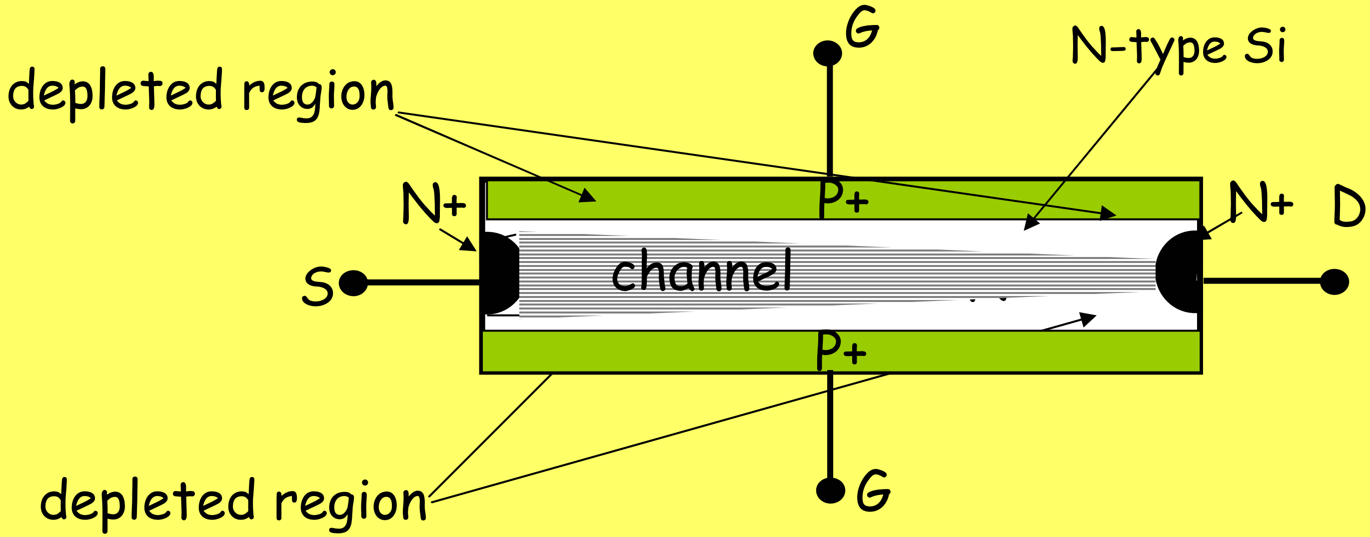
Noise in field-effect devices in the high frequency region. In field-effect devices the signal gain is based on the modulation of the conductance of a channel. The presence of a channel of finite conductivity suggests the noise in the channel current to be of thermal nature. Such a conclusion is presented in a fundamental paper by A. van der Ziel in 1962 (*). At that time the nature of the noise on field-effect devices was still an open question, as apparent from this sentence in van der Ziel's paper:

It has been suggested (**) that the observed noise can be interpreted as suppressed shot noise. There is no physical basis for such a suggestion. For evidently the field-effect transistor operated on the principle of true conductance modulation, as Shockley's theory indicates. Generally one associates thermal noise with a true conductance and not shot noise. It is hard to see how shot noise could ever be generated and, if generated, how it could be partly suppressed. As this paper indicates, the assumption of thermal noise allows straightforward explanation of the observed noise.

(*) A. van der Ziel – Thermal noise in Field-effect transistors – Proceedeings of the IRE, August 1962, pp 1808-1812

(**) P.O. Lauritzen – Field-effect transistors as low-noise amplifiers – Solid-sate Circuit Conference, Philadelphia, PA, February 14-16, 1962, Digest of Technical papers, pp 62-63, February, 1962.

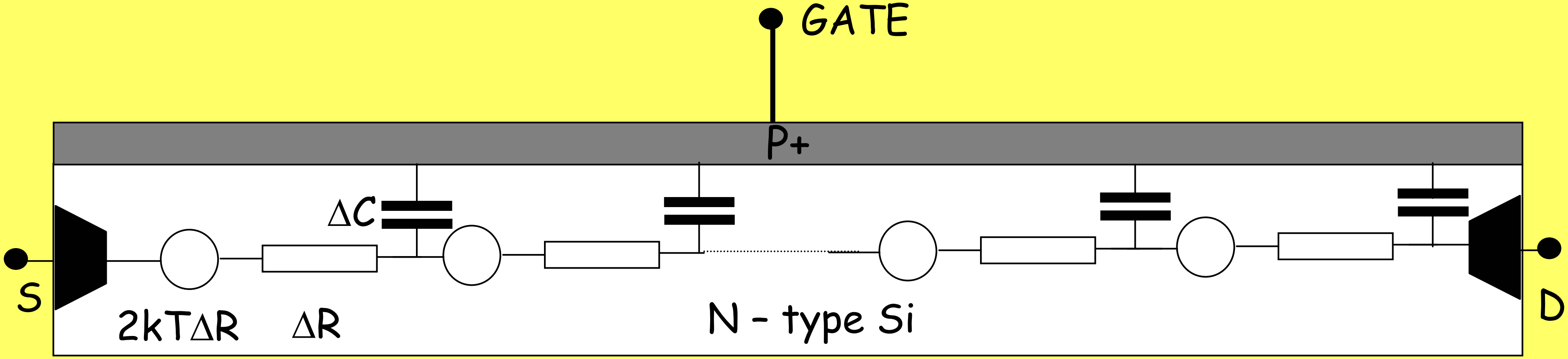
The quoted Van der Ziel paper, which even now is assumed as a fundamental analysis for the understanding of the channel thermal noise in all types of field-effect devices, was based on the gradual channel theory discussed by Shockley with reference to junction field-effect transistors (*).



The figure represents a double-gate JFET. In some applications, topside and backside gates can be short-circuited, while in some low-noise circuits, for reasons that will be clear later, it is preferable to keep them separated.

(*) W. Shocley – *A unipolar field-effect transistor – Proceedings of the I.R.E.*, ***40***,*1365 (1952).*

Channel Thermal Noise-Delay line model – This model applies to all devices, JFETs, MOSFETs, MESFETs where the high frequency noise is of thermal nature



The channel is modeled as a dissipative delay line, that is, a series connection of cells featuring an elementary resistance ΔR and an elementary capacitance ΔC toward the gate. The thermal noise of spectral power density 2kTΔR is associated with each resistor ΔR. The noise on the channel current is calculated on the basis of this model. The calculation shows that the channel thermal noise has a spectral power density $\langle i_{ch}^2 \rangle = 2kT\Gamma g_m$ where g_m is the transconductance and G the thermal noise coefficient. This model explains also the origin of a capacitive gate current correlated to the channel thermal noise. Such a current has an ω^2 frequency dependence.

For the sake of reference, the figure of the previous page represents a field-effect transistors of the junction type in silicon, the type of device assumed for the basic discussion in van der Ziel's paper. However, as already stated, **the delay-line model applies to all types of field-effect devices**. The channel thermal noise, evaluated on the basis of the delay-line model is expressed by the following spectral power density:

$$d\langle i_{ch}^2\rangle/df = 2kT\Gamma g_m$$

Where g_m è is the transconductance of the device and Γ a numerical coefficient of the order of unit. In an ideal junction field-effect transistor of an extremely long channel, Γ is 0.7. The values of the Γ coefficient in other types of field-effect devices will be discussed later and its dependence on the type of device, on its channel length and its working condition will be commented.

In the small signal equivalent circuit of the field-effect devices this noise is represented by a current source connected between source and drain.

Be careful not to be misled by the expression of the spectral power density of the channel thermal noise. Such a spectral density is proportional to the transconductance. ***However, don't rush to wrong conclusions!.*** The signal current determined by a voltage signal applied between gate and source is proportional to g_m^2.

As what matters is the signal-to-noise ratio, the correct conclusion is that the higher is the transconductance, the lower is the impact of channel thermal noise on the signal-to-noise ratio.

In the case of the bipolar transistor the previous conclusion can be rephrased by saying that:

The higher is the transistor current, the lower is the impact of the collector shot noise on the signal-to-noise ratio, Remember, indeed, that the transconductance of the bipolar transistor is qI_E/kT.

Remember

In the bipolar transistor the control voltage modulates the height of the barrier at the emitter-to-base junction. **Therefore the noise on the emitter current is shot noise.** This noise is transferred multiplied by α^2 into a collector current noise term and multiplied by $(1-\alpha)^2$ into a base current noise term. However, the division of the emitter current between collector and base is a random process. This means that there is an additional noise term, the so called partition noise on both collector and base current noise.

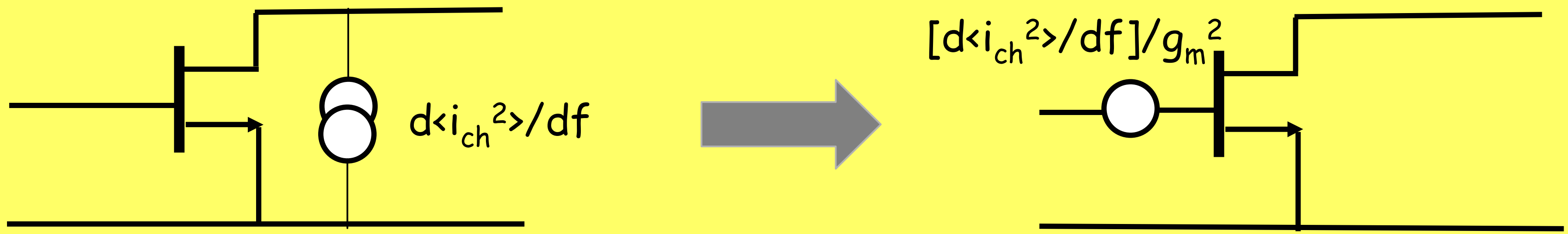
In all field-effect devices (JFET, MOSFET, GaAs MESFET) the control voltage modulates the conductivity of the channel. The high frequency noise is of thermal nature (see delay-line model at page 25).

From an equivalent point of view

The high frequency noise of all active devices can be represented by the circuit of next page where, for the sake of simplicity the same electrical symbol has been employed for all the devices

Referring the noise associated with the main current in the active device to its input.

In the previous noise analysis on the active devices the physics of the noise processes has led naturally to represent the noise with current sources connected at the output port of the device (drain-source or collector-emitter). The figure below shows how to transform the physical representation of noise into the equivalent one employing a voltage noise source in series with the control electrode.



The current source of spectral power density $d\langle i_{ch}^2\rangle/df$ is transformed into a voltage source in series with the control electrode by division by g_m^2. The new representation offers the advantage of a direct comparison of noise and signal at the input port of the device (gate-source or base-collector).

With the new representation the high frequency noise associated with the main current in the active devices can be expressed by a voltage source featuring the following spectral power density: $d\langle e_n^2\rangle/df = 2kT\Gamma/g_m$ in series with the control electrode (base or gate) and the following values of Γ:

- **Bipolar transistor** $\Gamma = 0.5$
- **JFET, long channel** $\Gamma = 0.66$
- **JFET, short channel** $\Gamma = 1$ and even slightly more
- **MOSFET, long channel**
  - **strong inversion** $\Gamma = 0.66$
  - **linear region** $\Gamma = 1$
  - **weak inversion** $\Gamma = 0.5$
- **MOSFET, short channel.** To hot electron effects are attributed values of Γ much larger than 1, up to 4, although values slightly above 1 are more reasonable.

NOISE IN THE LOW-FREQUENCY REGION

Although the noise in the high frequency region constitutes the dominant limitation of active devices in the amplification of signals from radiation detectors, the effects of low-frequency noise may be of importance in some applications. For this reason, the low-frequency noise mechanisms will be discussed here.

- Bipolar transistors are excellent devices as far as low-frequency noise behavior is concerned. Unfortunately, the shot noise associated with their base current restricts their use to very specific applications with capacitive sources like radiation detectors.

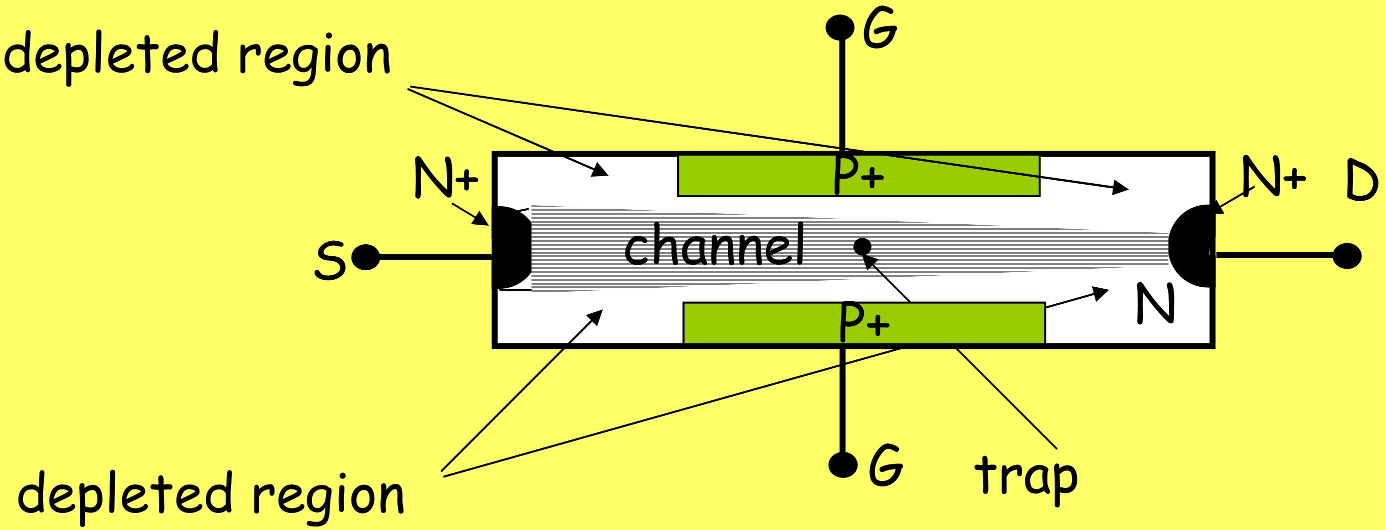
- The silicon junction field-effect transistor has the best low-frequency noise behavior of all field-effect devices.

- Enhancement-type MOSFETs have a large amount of 1/f-noise, which makes them unsuitable for detector applications where extremely good energy resolution is required.

- The low-frequency noise behavior of GaAs MESFET is also limited by 1/f noise.

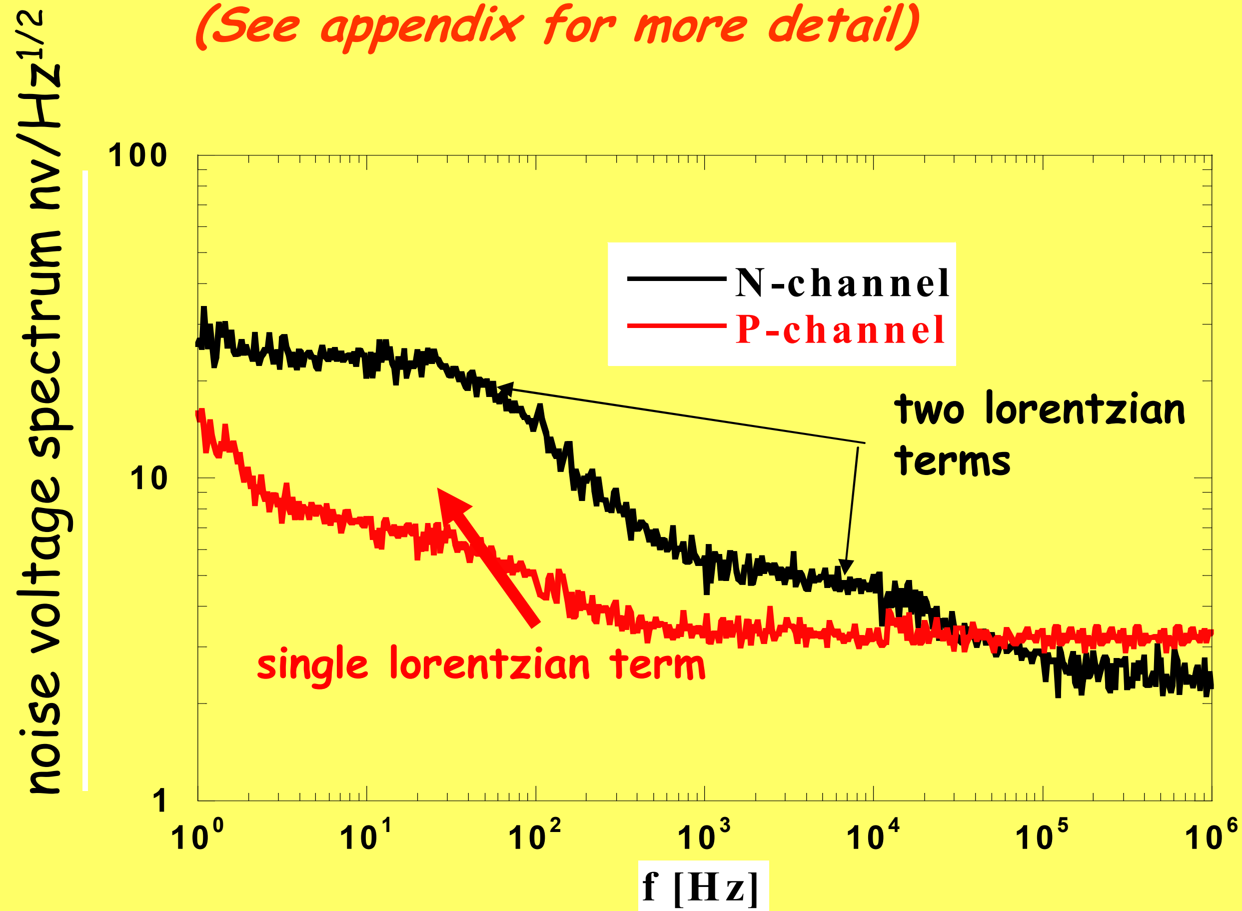
Low-frequency noise mechanism in Si JFET (See appendix for more detail)

Low frequency noise in si JFET is related to pointlike defects in the channel acting as traps for the mobile carriers. The noise due to a trap has a spectral power density of Lorentzian nature: $h_i / (1 + \omega^2 \tau_i^2)$, where τ_i is the time constant of the trap. Silicon JFETs usually have a comparatively small number of traps in the channel and their low-frequency noise contribution can be described as a sum of Lorentzian terms. In some applications it can be approximated as $1/\omega^2\tau^2$, where τ is the lowest Lorentzian time constant.



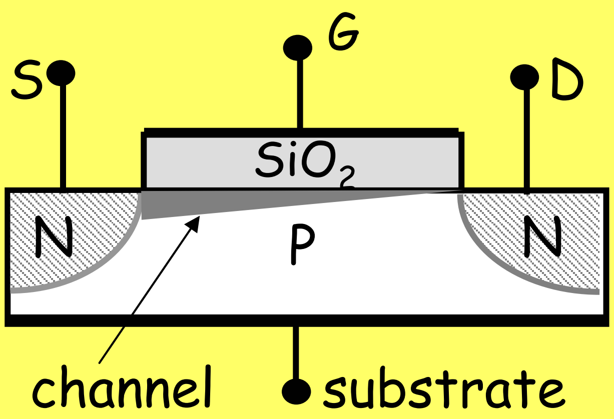
For a better understanding of lorentzian noise in JFETs, defects were artificially created in pairs of complementary JFETs belonging to a JFET-CMOS process provided by Fraunhofer Gesellschaft in Duisburg. (*)

(See appendix for more detail)



In this way the noise due to the Lorentzian terms becomes clearly evident.

Low frequency noise mechanism in enhancement-type MOSFETs



In enhancement-type, called also inversion-type MOSFETs because the channel sets up as an inversion in the bulk the conduction is of the surface type and takes place in the proximity of the insulator. Depending on the quality of the material, **a large number of traps may be present in the oxide**. Carriers may occupy empty traps at up to 20 Angstrom-depth into in the oxide by tunneling across the potential barrier at the interface Si-insulator. Besides, it must be taken into account that **traps are also present at the interface Si-insulator**. Owing to the large number of traps, the combination of the RTS converges to a spectral power density close to a 1/f frequency dependence.

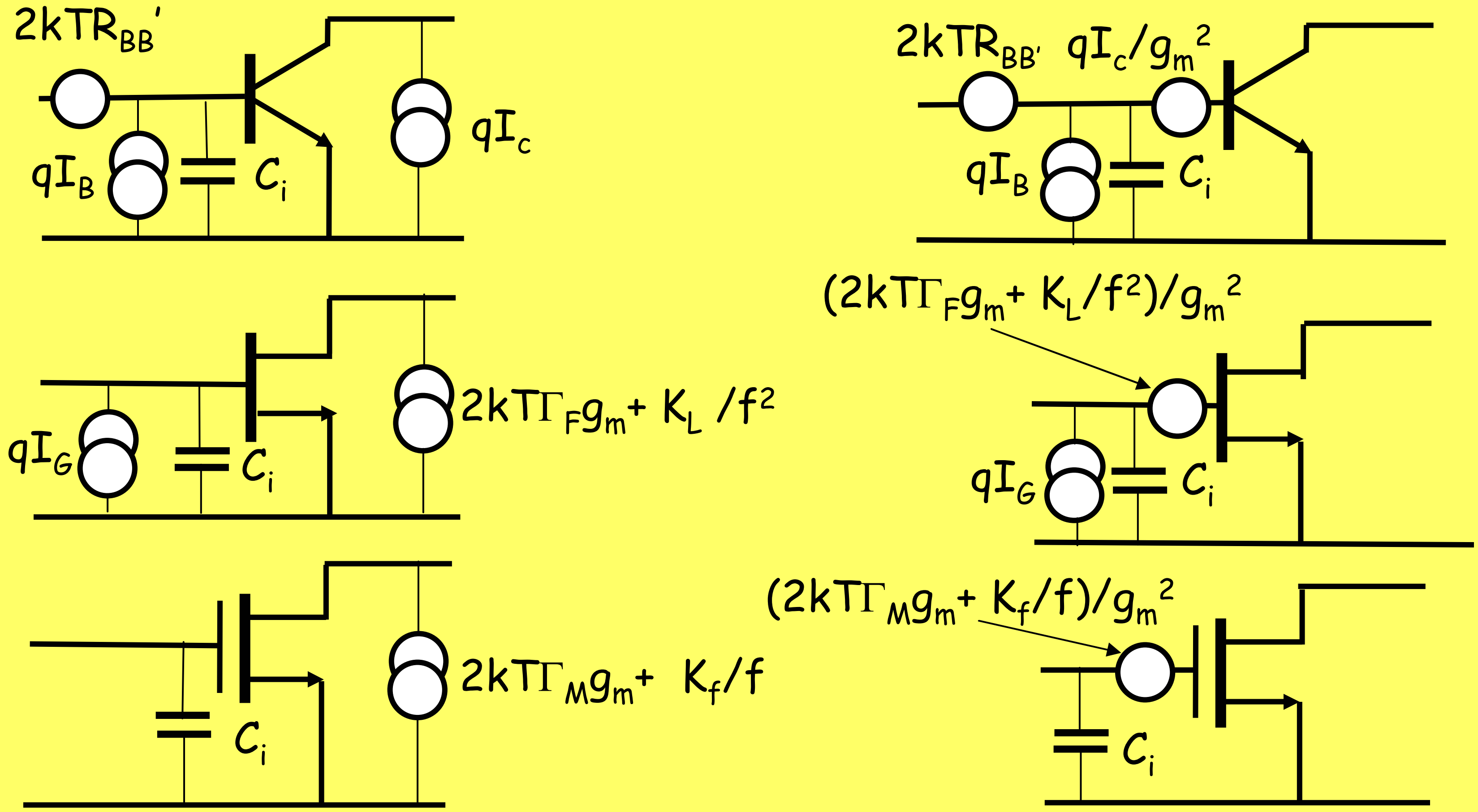
Spectral power densities of series (voltage) noise $S_M(f)$ in MOSFETS and $S_J(f)$ in JFETS:

$d\langle e_n^2\rangle/df = S_{0,M} + K_f / f^{a}$ $(a \sim 1)$ $\quad S_{0,M} = 2kT\Gamma_M / g_m$ **MOSFET**

$d\langle e_n^2\rangle/df = S_{0,J} + K_L / f^{2}$ $\quad S_{0,J} = 2kT\Gamma_F / g_m$ **JFET**

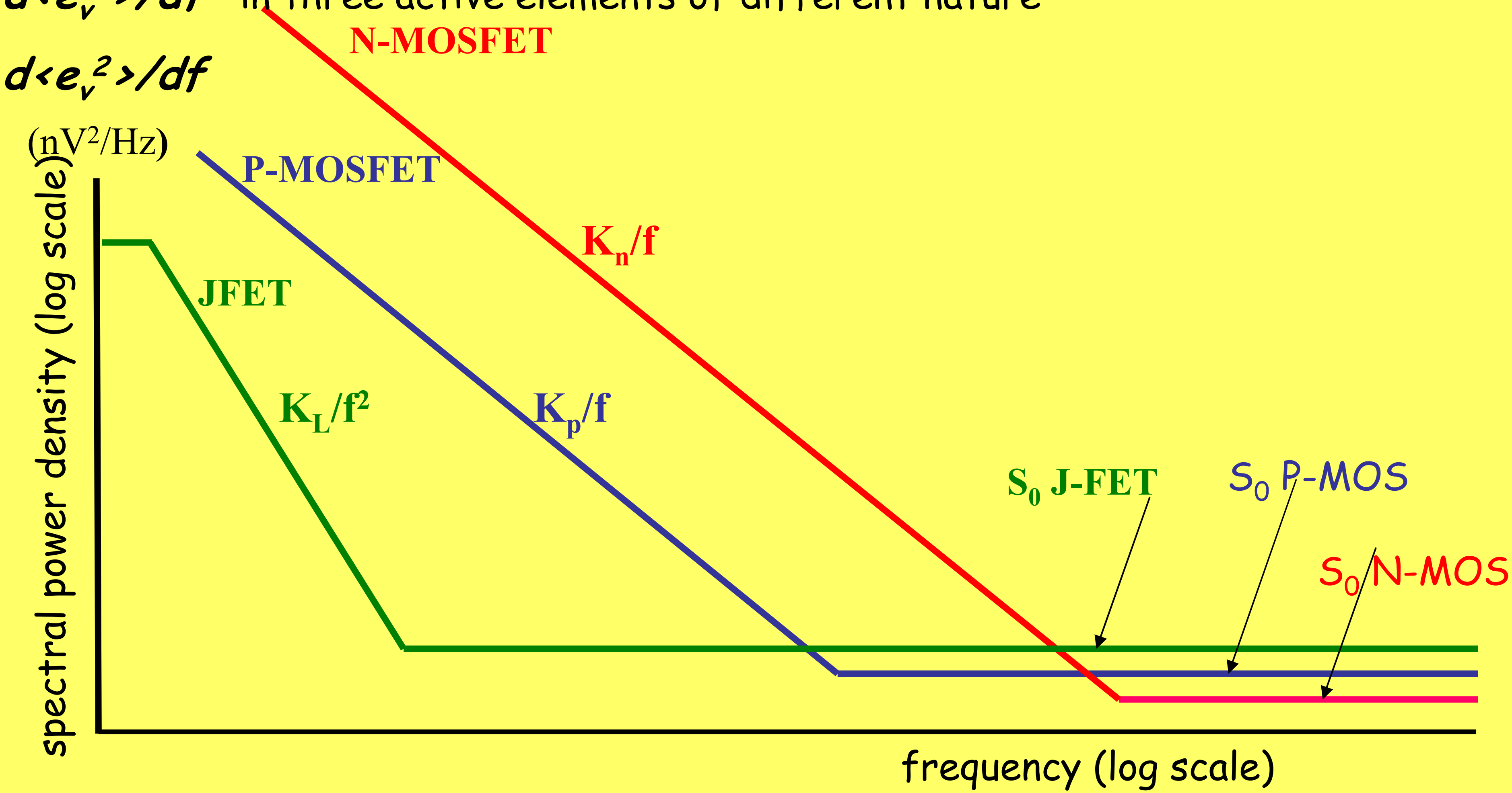
k is Boltzmann's constant,
T the absolute temperature,
g_m the transconductance,
Γ the coefficient of channel thermal noise
K_L the Lorentzian noise constant in the JFET.
K_f the 1/f – noise constant in the MOSFET

Noise representation by equivalent sources in the small-signal circuits

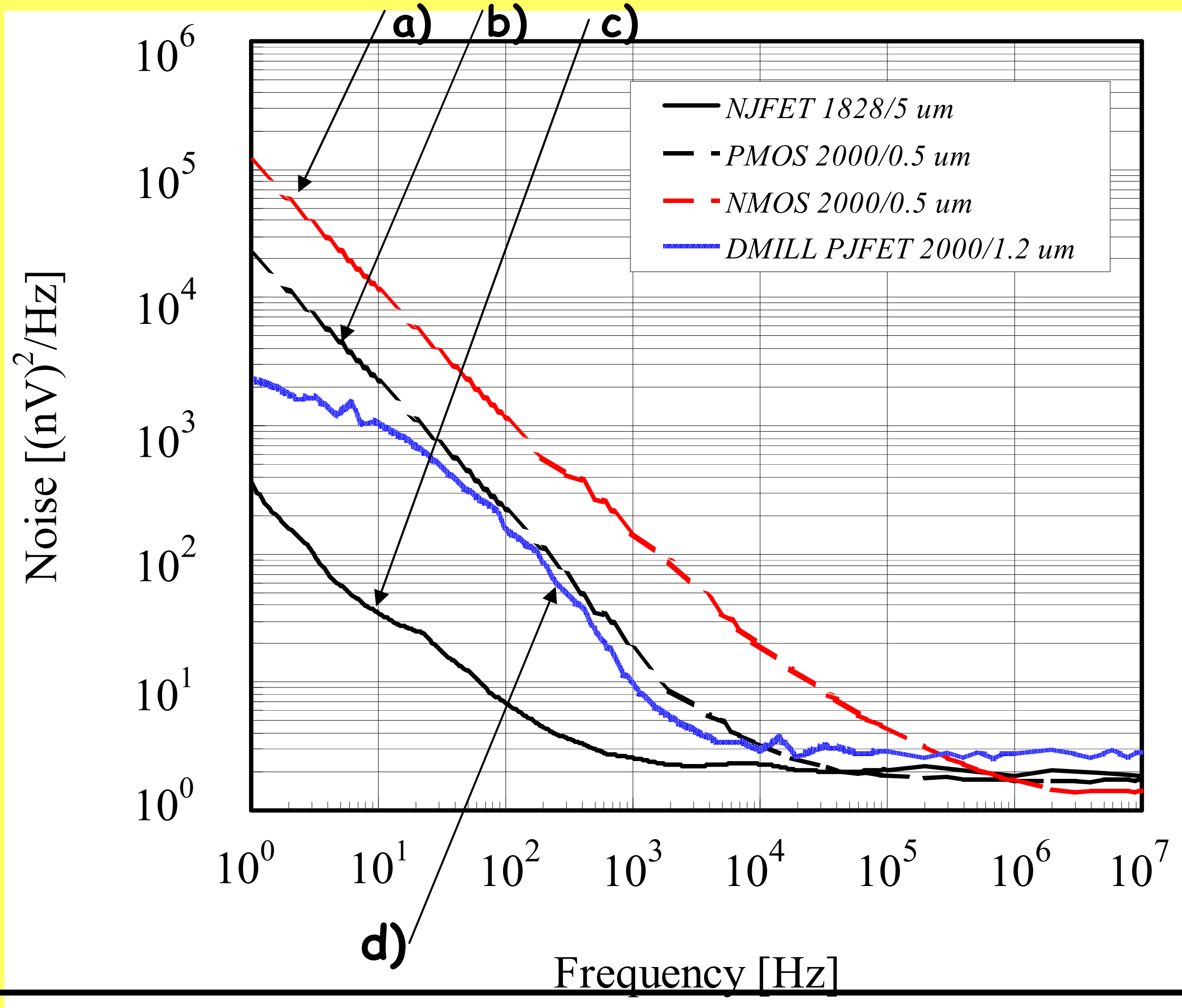


Remark: for the sake of simplicity, the source representing the gate-current noise at high frequencies, proportional to $\omega^2 C_{gs}^2$ is not represented here.

Qualitative comparison of the frequency dependence of the spectral power density $d\langle e_v^2\rangle/df$ in three active elements of different nature

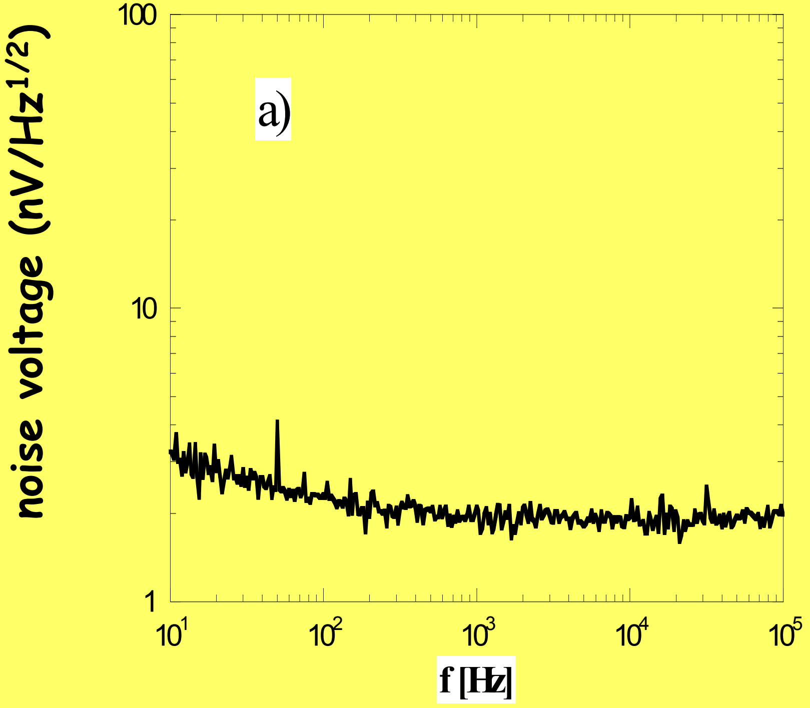


The following plots of the measured spectral power density confirm the qualitative behavior commented before.

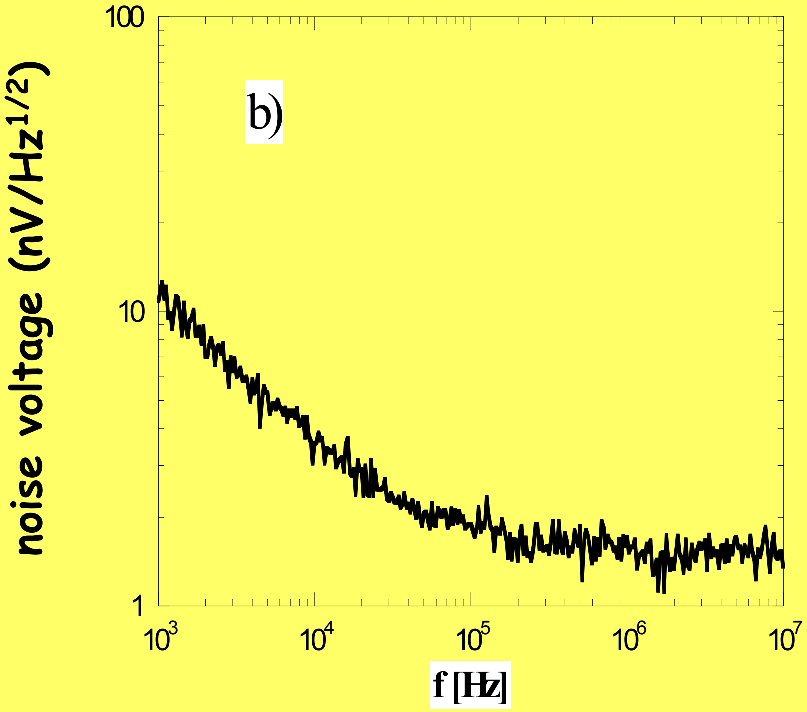


Frequency dependence of the $S_M(f)$, $S_J(f)$ spectral power densities for devices belonging to different monolithic processes: a) N-channel MOSFET belonging to a CMOS process featuring L_{min} = 0.25μm, t_{ox}=5nm, actual gate length L=0.5μm, drain current I_D = 0.5mA, input capacitance C_i = 6pF; b) P-channel MOSFET belonging to the same CMOS process as above, actual gate length L=0.5μm, drain current I_D=0.5mA, input capacitance C_i=6pF; c) N-channel JFET belonging to an all N-JFET monolithic process, actual gate length L=5μm, input capacitance C_i=10pF, drain current I_D=5mA; d) P-channel JFET belonging to DMILL BiCMOS JFET process, actual gate length L=1μm, input capacitance C_i=9pF, drain current I_D=1mA.

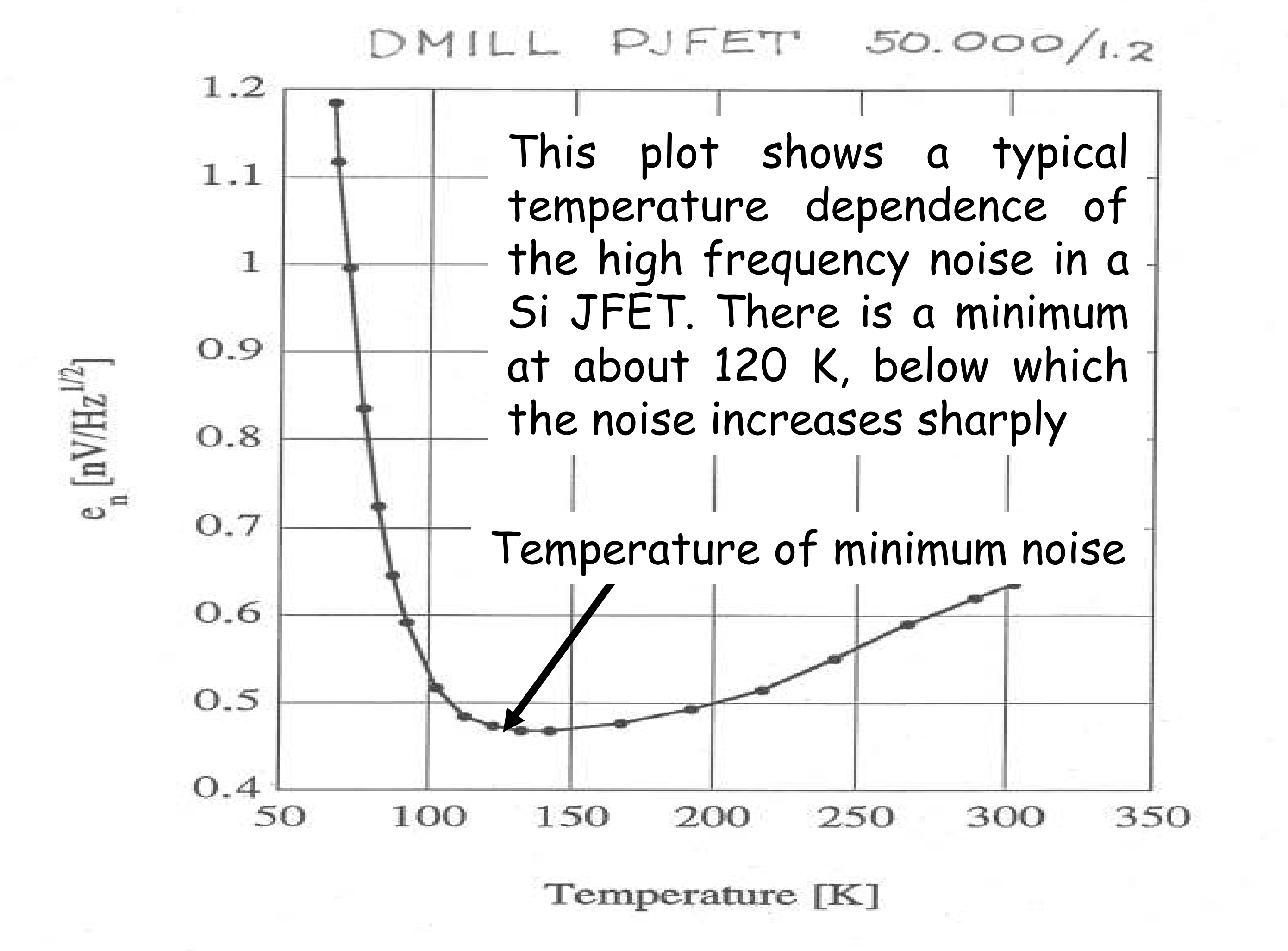
The two spectra shown below emphasize the reason why in certain frequency regions the JFET outperforms a PMOS, which in turn has less 1/f-noise than its N-channel complementary device.



N – channel JFET



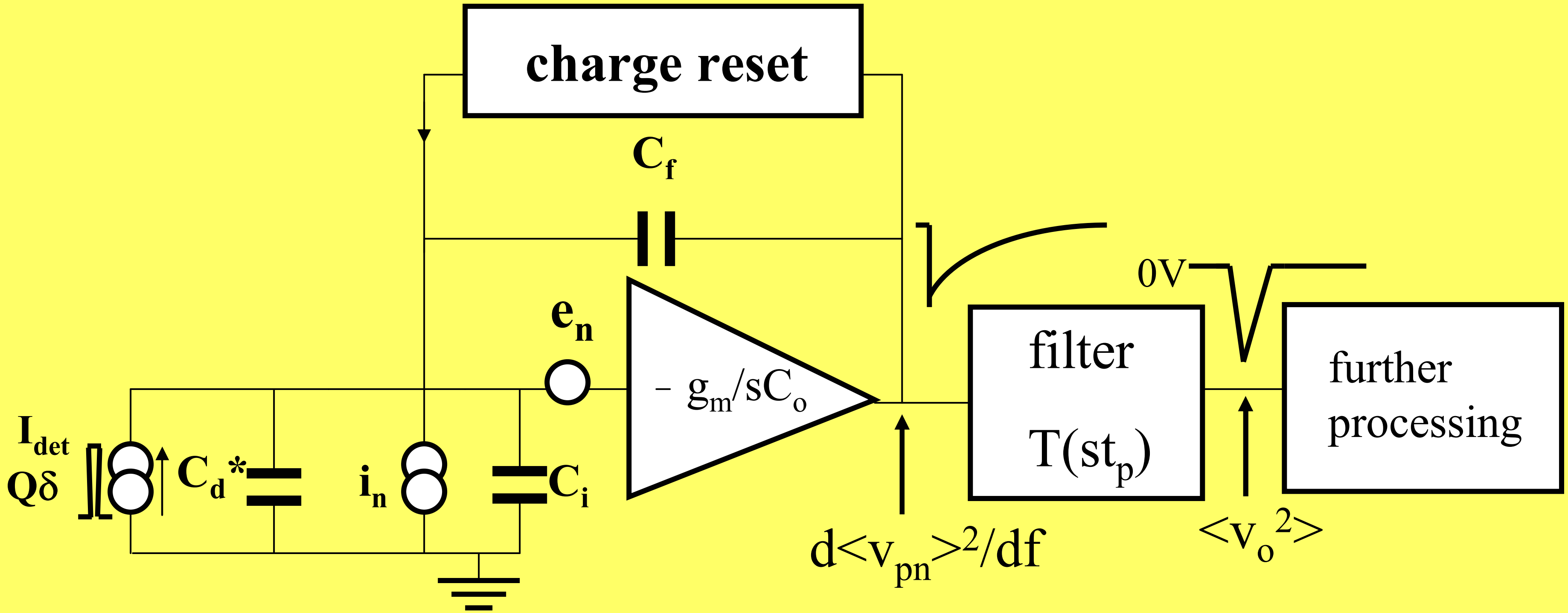
P channel submicron MOS



(See also appendix for more detail on temperature dependence of noise)

Detector charge measurement in the presence of noise

CHARGE MEASURING CHANNEL



Noise transformation in sinusoidal waves for gm/Co tending to infinity:

$$V_{pn}(\omega) = I_n(\omega)/J\omega C_f + E_n(\omega)(C_f+C_d^*+C_i)/C_f$$

In the previous figure:

$\mathbf{I_{det}}$ is the detector current signal

C_d* is the sum of detector capacitance and strays
C_f is the integration capacitance
C_i is the open-loop input capacitance of the preamplifier
$-g_m/sC_o$ is the open-loop transfer function of the active block in the preamplifier circuit
s is the complex variable

$\mathbf{e_n}$ is the voltage source which accounts for the input referred preamplifier noise (in a well designed circuit this is the noise associated with the drain or collector current in the front-end device). The noise described by $\mathbf{e_n}$ is referred to as ***series noise***.

$\mathbf{i_n}$ is the current source which account for all the noise sources that appear in parallel to the preamplifier input port. The noise described by $\mathbf{i_n}$ is referred to as ***parallel noise.***
It includes noise contributions from the detector, its bias network, and the charge-reset circuit, the noise in the input current of the front-end device as well as the noise arising from dielectric losses that may be present in the connectors, in the insulators that act as support for the leads terminating at the preamplifier input and in the detector itself.

The concept of Equivalent Noise Charge (ENC)

Equivalent Noise Charge (ENC) is the value of charge that injected across the detector capacitance by a δ-like pulse produces at the output of the shaping amplifier a signal whose amplitude equals the output r.m.s. noise, i.e. is the amount of charge that makes the S/N ratio equal to 1.
ENC is evaluated in the following way: from the schematic of page 43, the rms noise at the filter output is determined as:

$$<v_o^2> = \int_{-00}^{+00} (d<v_{pn}^2>/df\,)\, |T(j\omega t_p)|^2 df$$

The peak of the signal at the filter output is then evaluated. The gain of the system is supposed to be normalized, such that the signal peak be $Q/C_{f.}$
Then:

$$ENC^2 = C_{f.}^2 <v_o^2>$$

The calculation of the integral must be carried out by observing that, according to the equation of page 43,

$$d\langle v_{pn}^2\rangle/df = d\,\langle i_n^2\rangle/df(1/\omega^2 C_f^2) + d\langle e_n^2\rangle/df\,)[(C_f+C_d^*+C_i)/C_f]^2 \qquad (\omega = 2\pi f)$$

The expressions of $d\langle e_n^2\rangle/df$ for the MOSFET and the JFET can be found at page 36.

The complete expression of $d\,\langle i_n^2\rangle/df$ is given at page 48. Introduce then the expression of $d\langle v_{pn}^2\rangle/df$ into the integral and change variable by putting $x = \omega t_p$.

The resulting squared equivalent noise charge, ENC^2, depends on the peaking time t_p and contains three numerical coefficients defined at page 52, that are the characteristic parameters of the filter for the noise components at its input that depend on f like f^0, f^{-1}, f^{-2}.

Suggestion for noise calculation. Introduce the expression above of $d\langle v_{pn}^2\rangle/df$ into the integral of page 44 and change variable in the integrals by putting $\omega t_p = x$ remembering that $\omega = 2\pi f$. You end – up to expressions that contain the three integrals of page 52. These integrals depend on the filter employed.

CONTRIBUTIONS TO THE EQUIVALENT NOISE CHARGE ENC_M IN MOSFETS AND ENC_J IN JFETS BROUGHT ABOUT BY THE VOLTAGE SOURCE e_n

$$ENC_M^2 = (C_D^{**} + C_i)^2 [A_1 S_{0,M} / t_p + 2\pi K_f A_2]$$

$$ENC_J^2 = (C_D^{**} + C_i)^2 [A_1 S_{0,J} / t_p + K_L A_3 t_p]$$

C_D^{**} is the sum of all capacitances shunting the preamplifier input port, $C_D^{**} = C_D^{*} + C_f$

C_i is the input capacitance of the front-end device

A_1, A_2, A_3 are the filter coefficients for the f^{0}, f^{-1}, f^{-2} noise spectral densities

t_p is the peaking time in the δ-response of the analog channel

ENC CONTRIBUTIONS DUE TO THE PARALLEL NOISE TERMS INCLUDED IN THE CURRENT SOURCE i_n

The spectral power density of the noise source i_n is expressed by the following equation:

$$d\langle i_n^2\rangle/df = 2kT/Rp + q(I_L + I_D) + c_1/f/ + c_2/f/^{-1}$$

1 2 3 4

1 Represents the thermal noise associated with the bias network of detector and with the charge reset circuit if this is of resistive nature.

2 Is the shot noise associated with the detector leakage current and with the preamplifier input current.

3 Is the noise related to dielectric losses.

4 Is the parallel 1/f - noise

Remark: The physical position of the source i_n is the same as the detector's. Therefore, the ENC contribution from the detector current noise is independent of the detector capacitance.

CORRESPONDENCE BETWEEN THE ω DEPENCENCE OF di_n^2/df AND THE ω – DEPENDENCE OF dv_n^2/df

	di_n^2/df	dv_n^2/df
1,2	ω^0	ω^{-2}
3	$\lvert\omega\rvert$	$\lvert\omega\rvert^{-1}$
4	$\lvert\omega\rvert^{-1}$	$\lvert\omega\rvert^{-3}$

The interesting aspect related to type 4 of spectral density is that its ENC contribution would diverge unless a bipolar shaping is employed.

CORRESPONDENCE BETWEEN THE ω DEPENCENCE OF $d\langle v_{pn}^2\rangle/df$ AND THE t_p – DEPENDENCE OF ENC^2

$d\langle v_{pn}^2\rangle/df$	ENC^2
$\|\omega\|^{-1}$	t_p^{0}
ω^0	t_p^{-1}
ω^{-2}	t_p^{+1}

TOTAL ENC²

The total ENC² is obtained in this way. Recall the expressions of the equivalent noise charge due to the series noise sources:

$$ENC_M^2 = (C_D^{**} + C_i)^2 [A_1 (2kT\Gamma_M / g_m) / t_p + 2\pi K_f A_2]$$

$$ENC_J^2 = (C_D^{**} + C_i)^2 [A_1 (2kT\Gamma_F / g_m) + K_L A_3 t_p]$$

To either ENC_M^2 or ENC_J^2 the contribution brought about by i_n must be added :

$$ENC_{PAR}^2 = (2kT/R_p + q(I_L + I_D))\, A_3 t_p + c_1 A_2 / 2\pi + \text{potentially divergent term}$$

The following expressions of the total ENC² are obtained

Final form of ENC^2_{tot} *for the MOSFET:*

$$ENC^2_{tot} = (C_D{**}+ C_i)^2 A_1 (2kT\Gamma_M / g_m)/t_p + (2\pi K_f (C_D{*}+ C_f + C_i)^2 + c_1 / 2\pi) A_2$$

$$+ (2kT/R_p + q(I_L + I_D)) A_3 t_p$$

Final form of ENC^2_{tot} *for the JFET:*

$$ENC_J^2 = (C_D{**}+ C_i)^2 [A_1 (2kT\Gamma_F / g_m) / t_p + K_L A_3 t_p] + (2kT/R_p + q(I_L + I_D)) A_3 t_p + c_1 A_2 / 2\pi$$

A_1, A_2, A_3 *are the coefficients of the shaper for noise terms with* ω*- dependence of their spectral power density respectively of the type:* ω^0, ω^{-1}, ω^{-2}.

COEFFICIENTS

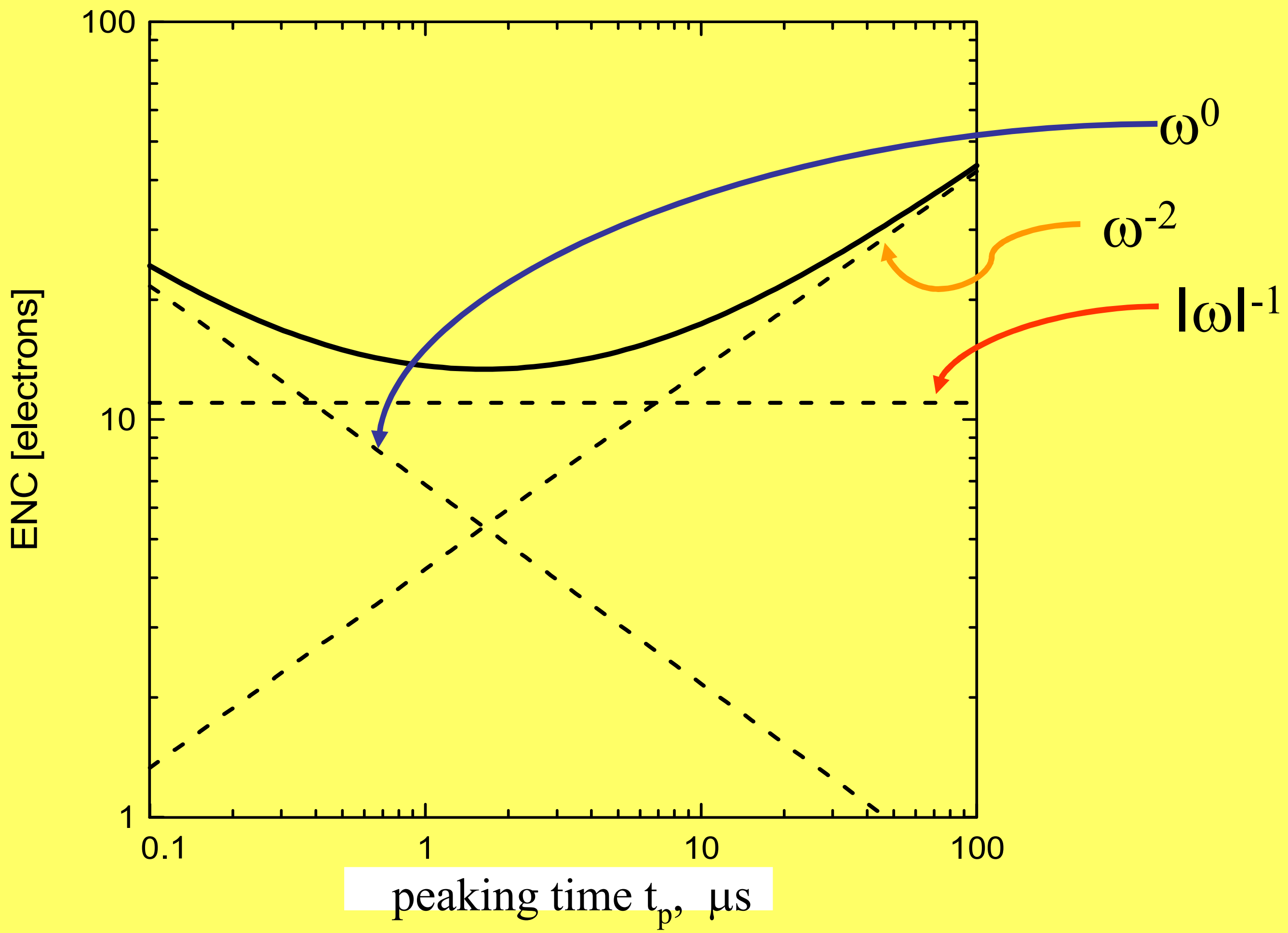
$$A_1 = (2\pi)^{-1} \int_{-\infty}^{+\infty} |T(x)|^2 dx$$

$$A_2 = (2\pi)^{-1} \int_{-\infty}^{+\infty} |x|^{-1} |T(x)|^2 dx$$

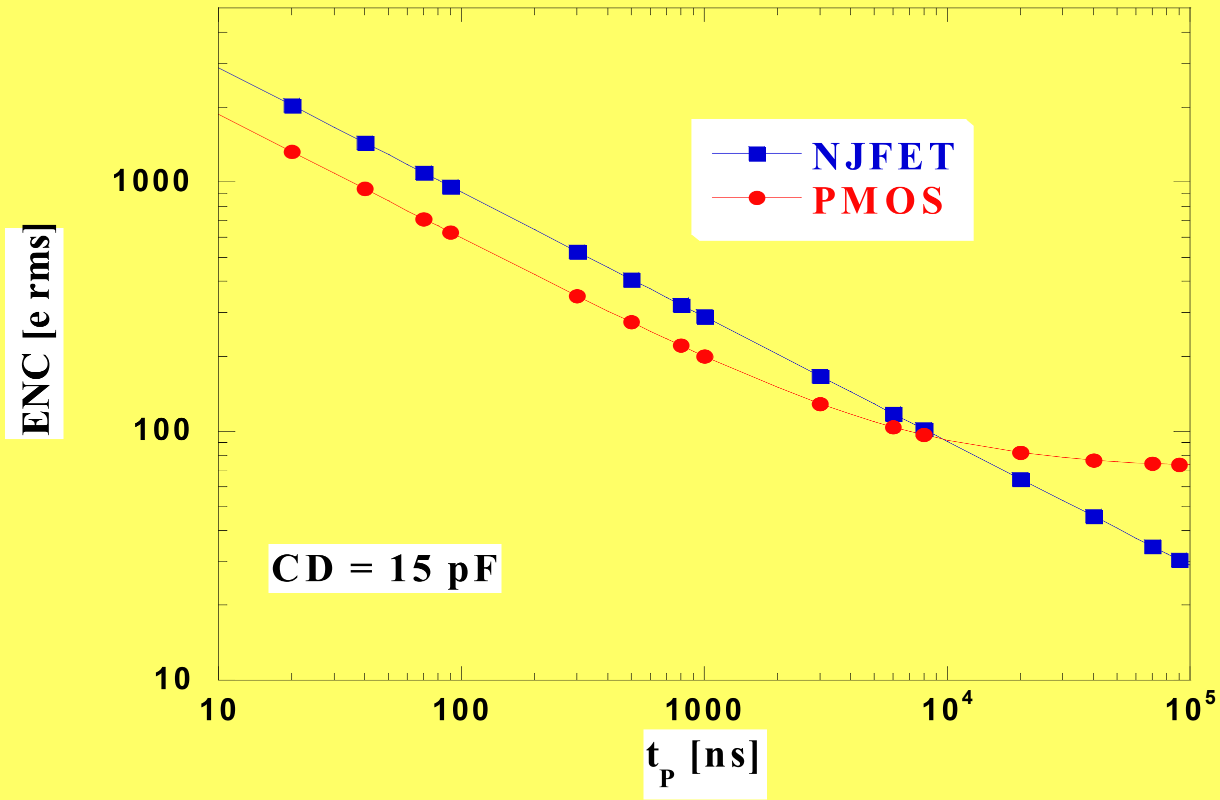
$$A_2 = (2\pi)^{-1} \int_{-\infty}^{+\infty} x^{-2} |T(x)|^2 dx$$

FILTERS

Shaping	h (t) Function	A_2	$\sqrt{A_1 A_3}$	$\frac{A_2}{\sqrt{A_1 A_3}}$	A_1	A_3	$\sqrt{\frac{A_1}{A_3}}$
indefinite cusp	$e^{t/\tau}$, $e^{-t/\tau}$, O	0.64 $\left(\frac{2}{\pi}\right)$	1	0.64	1	1	1
truncated cusp	$k = \tau'/\tau$; $-\tau'$, $-\tau$, O, τ, τ'; k=1	0.77	1.04	0.74	2.16	0.51	2.06
	k=2	0.70	1.01	0.69	1.31	0.78	1.30
	k=3	0.67	1	0.67	1.31	0.91	1.10
triangular	O, τ, 2τ	0.88 $\left(\frac{4}{\pi} \ln 2\right)$	1.15 $\left(\frac{2}{\sqrt{3}}\right)$	0.76	2	0.67 $\left(\frac{2}{3}\right)$	1.73
trapezoidal	O, τ, 2τ, 3τ	1.38	1.83	0.76	2	1.67	1.09
piecewise parabolic	O, τ, 2τ	1.15	1.43	0.80	2.67	0.77	1.86
sinusoidal lobe	O, τ, 2τ	1.22	1.57	0.78	2.47	1	1.57
RC-CR	O, τ	1.18	1.85	0.64	1.85	1.85	1
semigaussian (n = 4)	O, 4τ	1.04	1.35	0.77	0.51	3.58	0.38
gaussian	$e^{-1/2(t/\tau)^2}$, O	1	1.26	0.79	0.89	1.77	0.71
clipped approximate integrator	O, 0.5τ	0.85	1.34	0.63	2.54	0.71	1.89
bipolar triangular	O, τ, 2τ	2	2.31	0.87	4	1.33	1.73


100
10
1
0.1
1
10
100
ENC [electrons]
peaking time t_p, μs
ω^0
ω^{-2}
$|\omega|^{-1}$

Comparison between the t_p - dependences of ENC for a JFET and a MOSFET. In a JFET ENC keeps decreasing as the peaking time is increased, while for the MOSFET it levels off at the noise floor determined by 1/f-noise.



Appendix on noise

Campbell's theorem

Campbell's theorem states that, calling λ the mean rate of arrival, average value and variance of the noise process are given by the following relationships:

$$\langle N(t)\rangle = \lambda \int_{-\infty}^{+\infty} f(t)\,dt \qquad \langle N(t)^2\rangle - \langle N(t)\rangle^2 = \lambda \int_{-\infty}^{+\infty} f^2(t)\,dt$$

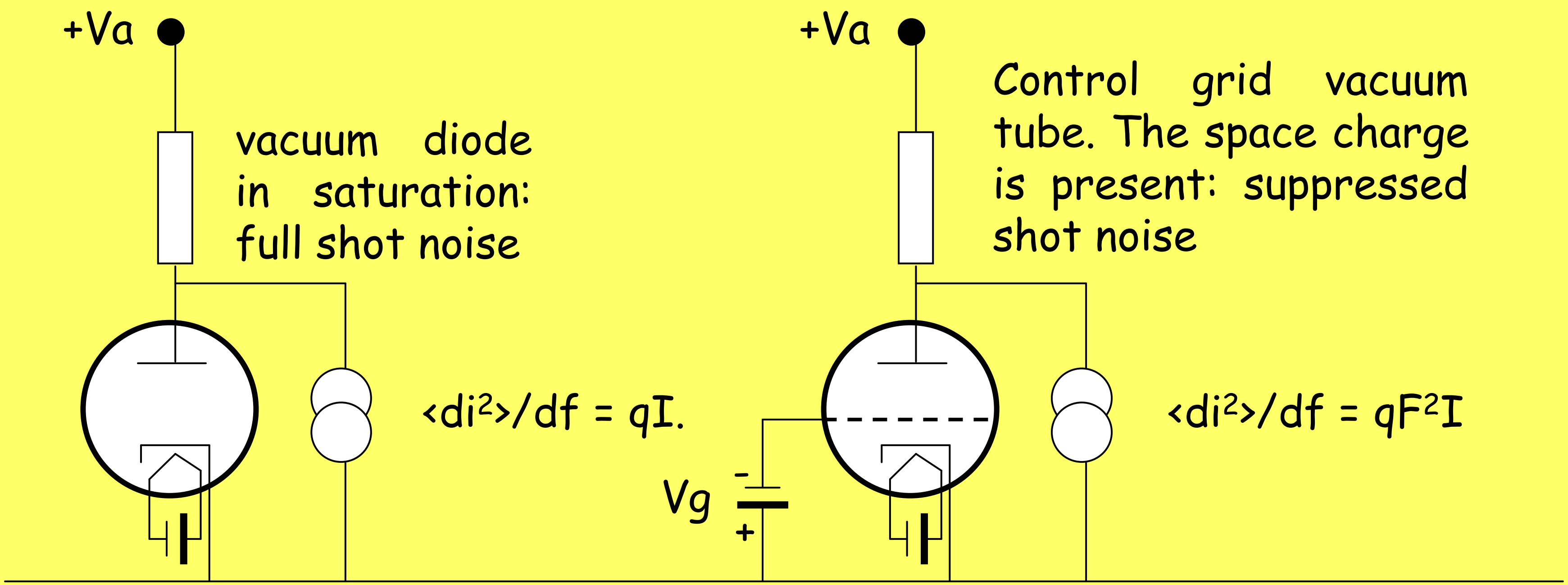
If Parseval's theorem is applied to the second integral, an interesting result is reached, that is, **the spectral power density of the noise process obtained in this way is calculated from the knowledge of λ e and of the Fourier transform *F(f)* di f(t):**

$$\langle N(t)^2\rangle - \langle N(t)\rangle^2 = \lambda \int_{-\infty}^{+\infty} f^2(t)\,dt = \lambda \int_{-\infty}^{+\infty} \left|\Im(f)\right|^2 df = \int_{-\infty}^{+\infty} S_N(f)\,df$$

where $S_N(f)$ is the spectral power density ot the N(t) process and $\Im(f)$ the Fourier transform di f(t). According to the last relationship:

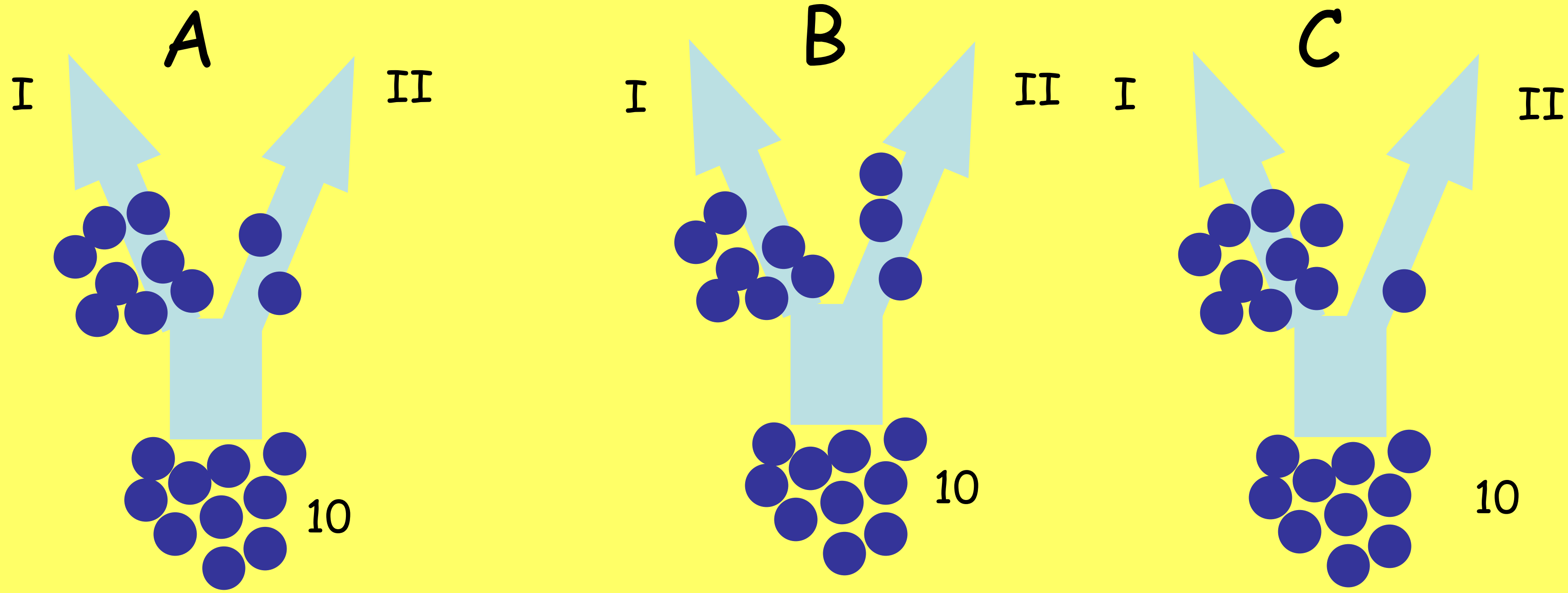
$$S_N(f) = \lambda \left|\Im(f)\right|^2$$

Shot noise and partition noise



The shot noise is related to the granular structure of electricity, and for this reason, for instance, the value of the elementary charge q can be determined from accurate measurements of shot noise associated with a reliable and stable source. **Shot noise appears when an energy selection, in our example actuated by the potential barrier, is made on a gas of electricity carrier. The elementary events consisting in carriers jumping beyond the potential barrier are randomly distributed in time and this randomness reflects on the fluctuations in the current at the collecting electrode.**

Partition noise. Consider a bunch of N charge carriers, electrons or holes, that constitutes a current $qN/\Delta t$, Δt being the duration of the bunch. Suppose that the N carriers split along two paths I and II, according to a given partition ratio. Let hN, h<1 the number of carriers following path I. Then the number of carriers following path II is (1-h)N. Figure A shows, as an example, the case of N = 10 and h=0.8. Out of the 10 carriers of the initial bunch, 8 follow path I and 2 path II.



If the partition of the initial bunch between paths I and II is rigid and the launch of the N carriers is repeated over and over again, for each bunch the splitting will occur in the same way, hN carriers following path I and (1-h) carriers following path II. So, in the example of case A in the previous figure, 8 carriers out of ten will follow I and 2 out of ten will follow II.

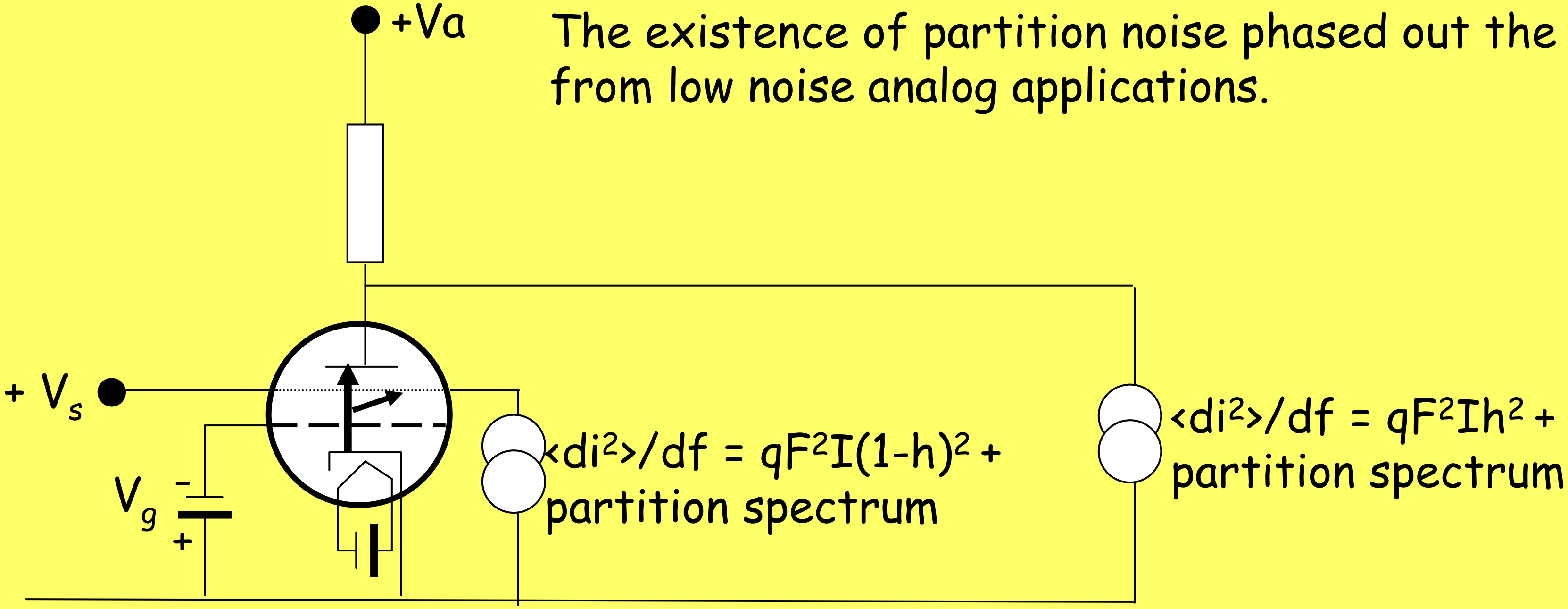
If instead the partition occurs at random, the number of carriers traveling along I and II will fluctuate from launch to launch. This situation is shown to occur in cases B and C of the previous figure, where 7 carriers instead of 8 go into I and 3 instead of 2 go into II (case B), while in case C, 9 carriers end up to I and 1 ends up to II. The numbers of carriers in I and II are now random variables and their fluctuations are totally correlated, as shown in the previous figure, where an increase over the expected value hN in the number of carriers in I corresponds to an equal decrease in the number of carriers in II. The statistics governing the random partition is Bernoulli's and assuming h to represent now the probability of a carrier going into I, the variance in the number of carriers along I and II is

$$h(1-h)$$

A current flowing into the junction I and II and splitting in a random way exhibits accordingly a noise called partition noise.

Partition noise was observed the first time in vacuum tetrodes, where the splitting of the cathode current occurs between the anode and the screen grid, an electrode introduced in order to shield to reduce the effect of the voltage swings at the anode on the electric field in the proximity of the cathode.

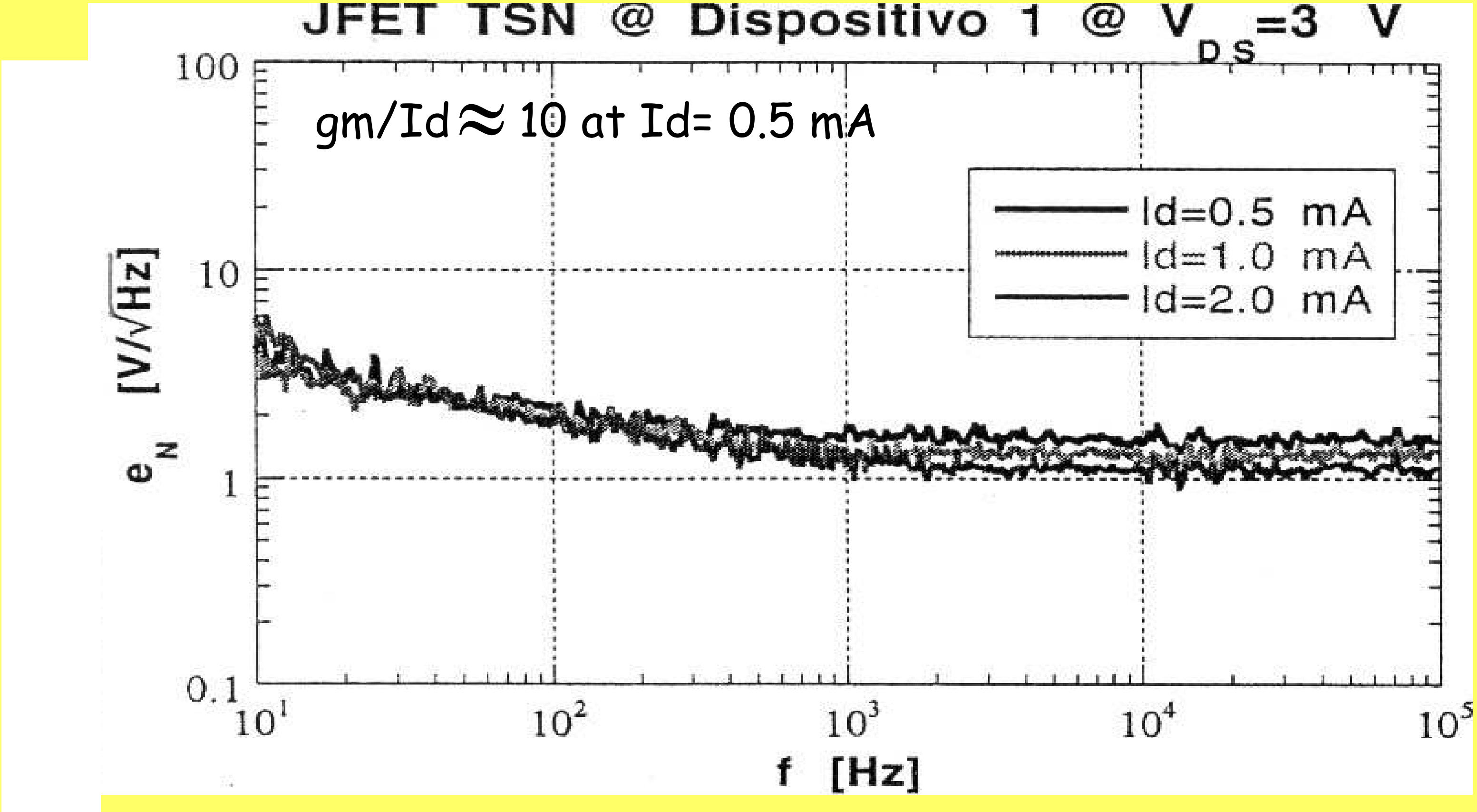
The existence of partition noise phased out the tetrode from low noise analog applications.

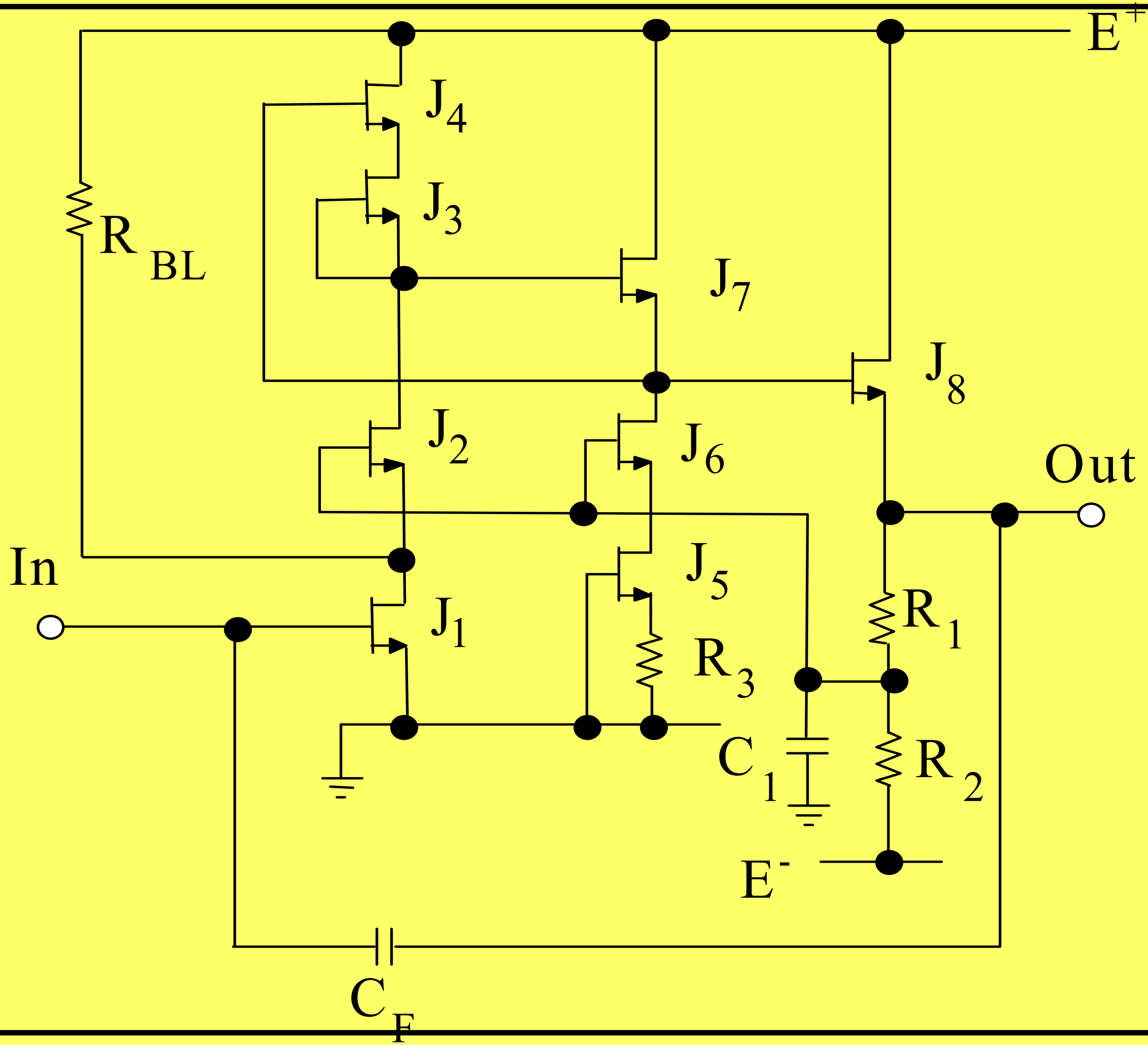


Vacuum tubes are certainly outside the scope of the actual lecture. However, the knowledge of the partition noise will help the understanding of noise in bipolar transistors, where the emitter current splits between base and collector.

Noise behavior of JFETs

This is for the purpose of showing an example of the limits achievable with one of the best discrete JFETs. In this specimen, at 0.5 mA I_D the gate referred noise remains below 2 nV/Hz$^{1/2}$ from 100 Hz on, quite a remarkable behavior.





This circuit is entirely based upon N-channel JFETs belonging to a process of outstanding noise performances.

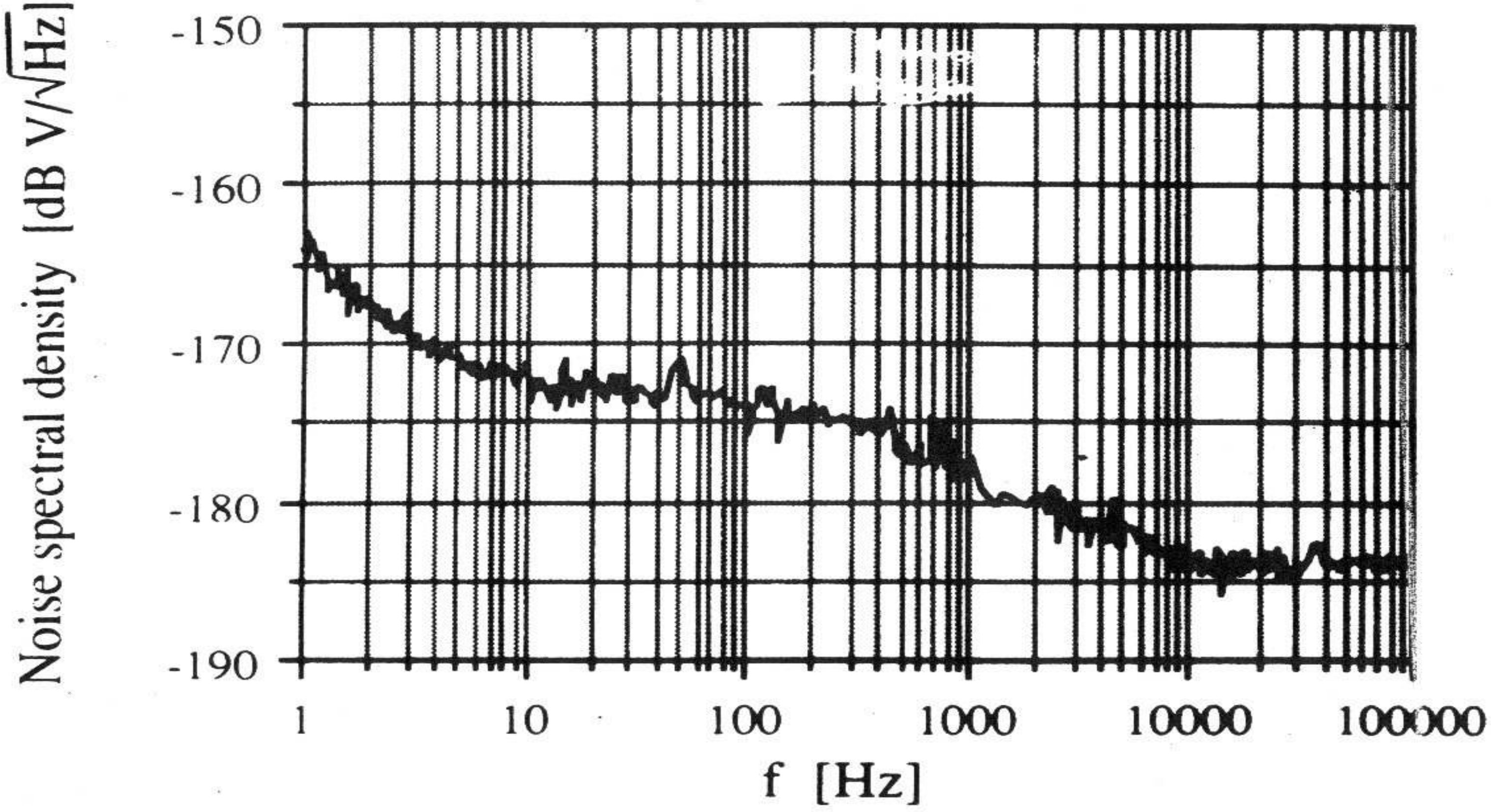
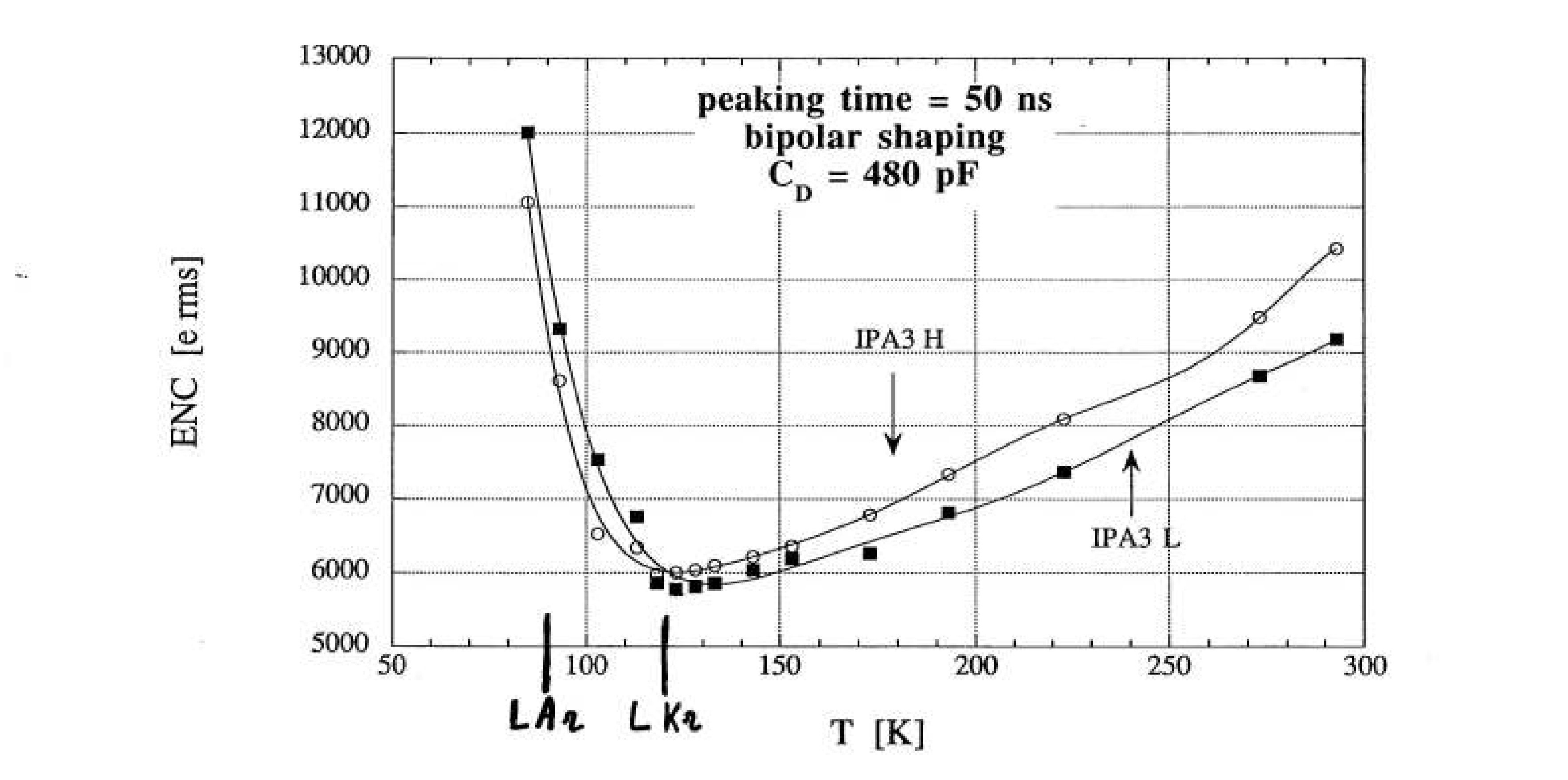
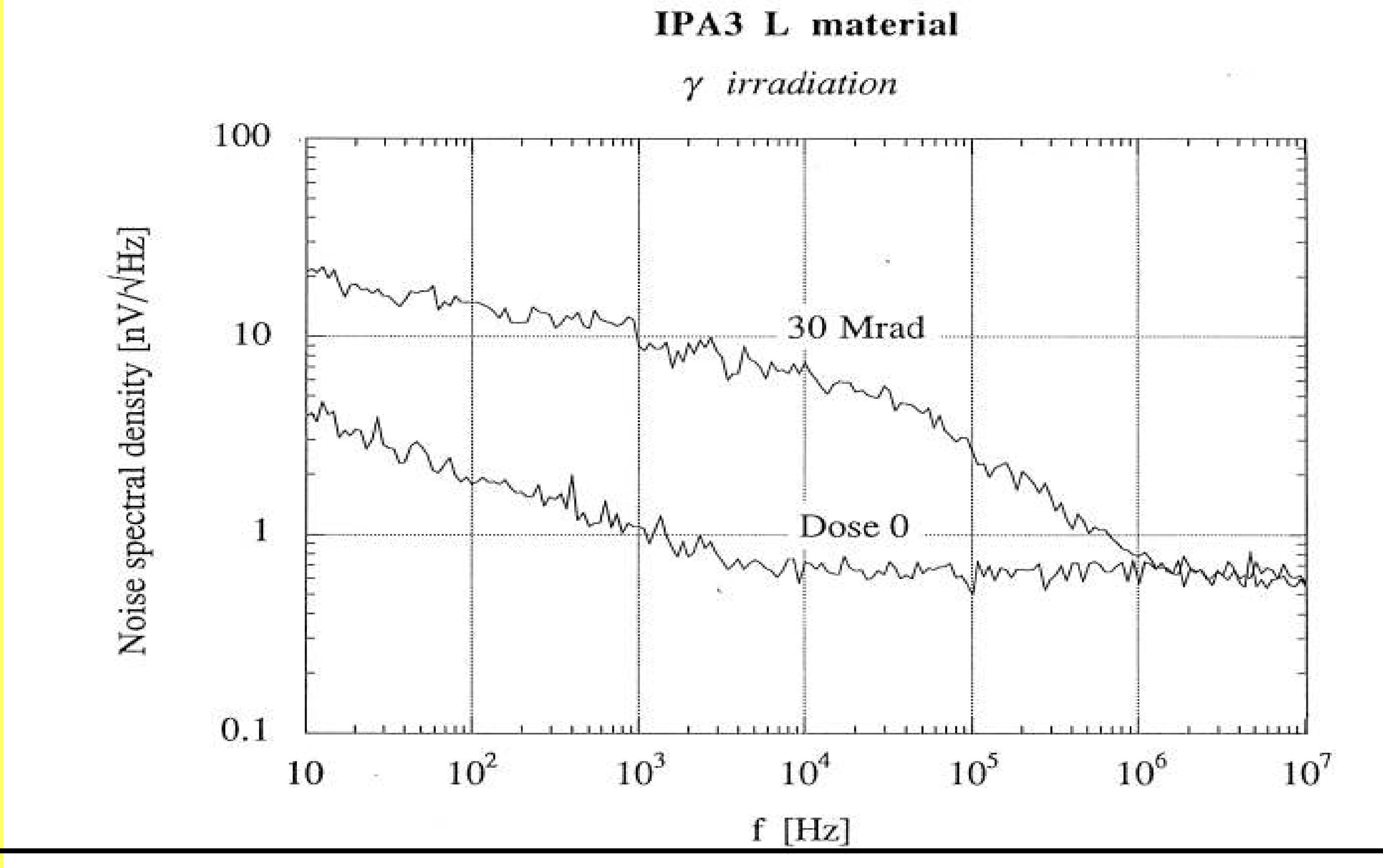


Fig. 4. Spectral density of the noise voltage referred to the preamplifier input as a function of frequency (0 dB corresponds to 1 $\mathrm{V\,Hz^{-1/2}}$. At $f > 10^4$ Hz $e_n \cong 0.63\ \mathrm{nV\,Hz^{-1/2}}$.

Temperature dependence of the noise charge for the All-JFET integrator based on the buried layer approach. The curves refer to two circuits of different channel doping (L and H materials). The attempt was to shift the condition at which the minimum occurs to lower temperatures.



Example of gamma-ray induced defects in the Buried-Layer JFET preamplifier and appearance of new Lorentzian terms.



Circuits featuring highly reproducible noise performances. The reliability of the noise performances in a process is an aspect which is frequently disregarded. It is impossible to discuss this issue in detail here. Just as an example, it is shown here how reliable in their noise behavior the integrators based on the JFET Buried Layer process are.